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ELEMENTARY ARITHMETIC

BY

DEFOREST A. PRESTON
PRINCIPAL PUBLIC SCHOOL NO. 164, BROOKLYN, NEW YORK

AND

EDWARD L. STEVENS
ASSOCIATE CITY SUPERINTENDENT OF SCHOOLS
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PREFACE

THIS book is the first of a series which offers a complete course in arithmetic for elementary schools. It is intended for use by pupils of the third, fourth, and fifth years, and can be used with little or no preparatory development on the part of the teacher. Its scope is sufficient for the ordinary needs of life and is enough to make full compliance with the requirements of the compulsory education law as it is found in most states of the Union. The book is intended to meet conditions found in the classroom, and to appeal to the actual knowledge of the children.

While the work of the first two years of the elementary school does not require a text-book in the hands of pupils, because their ability to read is so limited that the work can better be presented by the teacher through the use of the blackboard, of the chart, and of objects, the suggestions of the introductory chapter will guide the work of preparation, during which time the book may be used in the hands of pupils to advantage.

Teachers will note the following features in this book:

The use of easy words in statements and problems. The discussion of theory after the child has learned the operation. The elimination of long explanations of processes.

Particular attention is called to the following:

The plan for subtraction. The condensation of the multiplication table. The method of reading exercises in division. Condensation of the various cases in multiplication and division of fractions into one case and one operation.

Whatever suggestions and explanations are found to be necessary are given to the pupil in the natural progress of his work, but these explanations are gathered into a summary after each topic has been considered.

Simple, direct, and easy problems are offered in abundance.

Oral and drill exercises in the abstract are intended to be left to the teacher in the class exercises.

The use of dollars and cents, and of concrete values, is early introduced. Analysis of problems is taught, and convenient forms for computation are suggested.

The teacher will find that each new step is closely dependent upon the preceding work, and the problems will be found to be carefully graded.

The form of figure arrangement is new and important. Figures in each of the four processes are used in the relative position found in actual computation, hence the child acquires notions of relation through visualization.

While the book is prepared for the regular grades of graded schools, it will be found to be well adapted for use in special classes or in ungraded schools.

NEW YORK, July, 1910.

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INTRODUCTION

FIRST AND SECOND YEARS

Note to Teacher. The work in number for the first year or two can be most satisfactorily presented by the use of objects, oral instruction, free use of the blackboard by teacher and pupils, simple stories and exercises in applied number, carefully planned busy work and directed motor activities, and by daily use of the drill cards, etc.

The exercises, therefore, in this chapter are not intended to show the full work of the pupils, but are given as suggestions and as outlines, which are to be expanded by the teacher and the pupils into such fullness as the natural progress of the class may require.

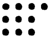
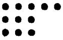
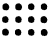
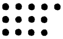
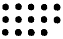
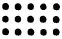
The exercises here outlined may also aid the teachers of the third year grades in reviewing the introductory work in number, and may help them to determine the class need for further instruction in any particular line.

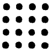
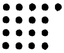
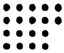
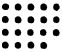
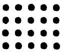
It is also hoped that the exercises of this chapter may prove helpful in aiding pupils, on entering school at an advanced age with considerable general knowledge of number, to pass rapidly over the introductory work and take up the regular exercises of the third year in a short time.

COUNTING

How many can you count?

Objects,	•	• •	•• •	•• ••	••• ••	••• •••	•••• •••	•••• ••••	••• ••• •••
Figures,	1	2	3	4	5	6	7	8	9
Names,	one	two	three	four	five	six	seven	eight	nine

Objects,						
Figures,	10	11	12	13	14	15
Names,	ten	eleven	twelve	thirteen	fourteen	fifteen

Objects,					
Figures,	16	17	18	19	20
Names,	sixteen	seventeen	eighteen	nineteen	twenty

Copy the following figures, and make pictures to show that you know what they mean:

7 5 8 3 9 4 2 12 15 17 6 16 18 19 20

Count the splints which are given you, and write the figure which will tell how many you have.

Show me four splints. Five splints. Eight splints. Ten splints.

Count out the number of objects which the figure written on the board shows, and place them on the end of your desk.

7 3 8 12

Write the figures on your papers which will answer these questions:

How many windows in the room?
How many marks did I make on the board?
How many books on my desk?
How many children in your row of seats?
How many boys in the class?

Watch me as I drop these pieces of chalk into the box, and count as I drop them, then write the figures on your papers that will tell me how many pieces I dropped into the box.

Count the steps I take in walking across the room, and tell me by your pencils how many I took.

Can you tell me some number stories like this? I have four cents. A dog has four feet.

ADDITION

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \underline{+1} & \underline{+1} & \underline{+1} & \underline{+1} & \underline{+1} & \underline{+1} & \underline{+1} & \underline{+1} & \underline{+1} \end{array}$$

Count out the number shown by one figure, and then count out the number shown by the other figure, and put the two piles together. How many in all? Count them.

Take splints in your hands, showing me the number in each hand, and put them together as we say, 2 and 1 are 3. Take 4 splints in one hand and 1 in the other, and put them together as we say, 4 and 1 are 5.

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \underline{+2} & \underline{+2} & \underline{+2} & \underline{+2} & \underline{+2} & \underline{+2} & \underline{+2} & \underline{+2} & \underline{+2} \end{array}$$

Put 2 splints with 2 other splints and count and see how many you have. Take 2 splints in one hand and 2 splints in the other hand, and as we put them together, say with me, 2 and 2 are 4. Copy the table which I have written on the board (the one printed above) and write the answers, counting splints to find the answer when you do not know it.

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ +3 & +3 & +3 & +3 & +3 & +3 & +3 & +3 & +3 \\ \hline \end{array}$$

Work out the answers to this table, and write them on your papers like this:

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ +3 & +3 & +3 & +3 & +3 & +3 & +3 & +3 & +3 \\ \hline 4 & & & 7 & 8 & & & 11 & 12 \end{array}$$

What are the answers to the other numbers?

Note to the Teacher. Drill cards showing the numbers in proper position, as shown above, should be prepared and used *daily* until the pupils can read them at sight. Have the pupils read each card both up and down as you direct; for example, tell them to read the cards "down," and they read (in the table of 3's), 1 and 3 are 4, 2 and 3 are 5, etc. If you ask them to read the cards "up," they read, 3 and 1 are 4, 3 and 2 are 5, 3 and 3 are 6, etc.

Work out the answers to this table, and write the table and the answers on your papers.

$$\begin{array}{cccccc} 4 & 5 & 6 & 7 & 8 & 9 \\ +4 & +4 & +4 & +4 & +4 & +4 \\ \hline \end{array}$$

Work out the answers to the table of 5's, and write the table and the answers on your paper.

$$\begin{array}{r}5\\+5\\\hline\end{array}\qquad\begin{array}{r}6\\+5\\\hline\end{array}\qquad\begin{array}{r}7\\+5\\\hline\end{array}\qquad\begin{array}{r}8\\+5\\\hline\end{array}\qquad\begin{array}{r}9\\+5\\\hline\end{array}$$

Work out the answers to each of the tables shown below, and write the tables and the answers on your paper.

$$\begin{array}{r}6\\+6\\\hline\end{array}\quad\begin{array}{r}7\\+6\\\hline\end{array}\quad\begin{array}{r}8\\+6\\\hline\end{array}\quad\begin{array}{r}9\\+6\\\hline\end{array}$$

$$\begin{array}{r}7\\+7\\\hline\end{array}\quad\begin{array}{r}8\\+7\\\hline\end{array}\quad\begin{array}{r}9\\+7\\\hline\end{array}$$

$$\begin{array}{r}8\\+8\\\hline\end{array}\quad\begin{array}{r}9\\+8\\\hline\end{array}$$

$$\begin{array}{r}9\\+9\\\hline\end{array}$$

If you learn to read these tables either way, these are all of the tables in addition you will ever have to learn.

Study the tables of 6's, 7's, 8's, and 9's, as they are the hardest to read at sight.

Make some "stories" about the tables. This is one: "James had 8 cents and his father gave him 7 cents, then he had 15 cents."

Another "story": "6 apples and 9 apples are 15 apples."

Reading and writing numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

201	202	203	204	205	206	207	208	209	210
301	412	513	614	725	836	912	918	971	987

Could you read the numbers below the dotted line without any help from your teacher? Can you read any number with three figures which your teacher writes on the board?

Write numbers as your teacher dictates them to you.

See if you can read the numbers rapidly, as the teacher points to some which she has written on the board.

Note to the Teacher. In this introductory work with numbers do not try to explain "place value" of figures, nor use the terms units, tens, hundreds. Teach the pupils to write 57 because that is the way to write it, just as you teach them to write "horse" in that way because it is *the* way to write it. Have them read numbers as a whole, just as you teach them to read words as a whole. Much time is wasted in arithmetic by too early explanations of why we do certain things as we do; and in the use of units, tens, and hundreds in explanations for reading and writing numbers; and in the plan of "carrying" in addition or "borrowing" in sub-

traction. Teach them the right way because it is the way other people do, and give the explanation later when the pupils will be able to benefit from its consideration.

ADDITION

Study the following exercises until you can read at once any card which the teacher shows you. The answers are not on the cards; but if you study them here, you will not need them on the cards.

8	18	28	38	48	58	68	78	88	98
+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3
11	21	31	41	51	61	71	81	91	101
9	19	29	39	49	59	69	79	89	99
+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3
12	22	32	42	52	62	72	82	92	102
7	17	27	37	47	57	67	77	87	97
+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4
11	21	31	41	51	61	71	81	91	101
8	18	28	38	48	58	68	78	88	98
+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4
12	22	32	42	52	62	72	82	92	102
9	19	29	39	49	59	69	79	89	99
+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4
13	23	33	43	53	63	73	83	93	103

Write out other tables like the ones just given, using 5, 6, 7, 8, and 9 as I have used 3 and 4 in the tables given. This work will help you all of your life, if you get it well done now, as every man likes to be able to add rapidly.

Tell me some number stories about the tables. Here are some, but you can tell me many more:

1. 27 oranges and 4 oranges are 31 oranges.
2. 48 cents and 3 cents are 51 cents.
3. 69 marbles and 4 marbles are 73 marbles.
4. A man had 18 sheep and bought 3 more, then he had 21 sheep.
5. A farmer had 57 acres of land and bought 4 acres more. He then had 61 acres.

Adding columns of figures.

(1)	(2)
324	342
132	132
+ 231	324
687	+ 132
	930

Write the numbers so that the right-hand column will be straight up and down.

Begin adding at the right-hand column.

The sum of the figures in the right-hand column is 10, but as that column will only take one figure, we write the naught, and add the 1 with the figures of the next column.

(3)	(4)	(5)	(6)	(7)	(8)	(9)
324	432	214	431	231	342	123
123	423	245	324	132	324	342
24	235	43	324	431	44	33
134	342	435	425	431	432	453

Write out many examples like the above for yourself, or for your seat mate. Use small figures in the numbers at first, until you can add well, then use the 6's, 7's, 8's, and 9's in hard examples like this:

Add 678, 769, 879, and 769.

SUBTRACTION

1. Frank had 5 cents and spent 2 cents for candy. How much had he left?

$$\begin{array}{r} 5 \\ -2 \\ \hline ? \end{array}$$

2 and ? make 5?

2. Six boys were playing and 3 went home. How many remained?

$$\begin{array}{r} 6 \\ -3 \\ \hline ? \end{array}$$

3 and ? make 6?

3. A man had 10 sheep and sold 4 of them. How many had he left?

$$\begin{array}{r} 10 \\ -4 \\ \hline ? \end{array}$$

4 and ? make 10?

4. A man has 20 sheep and sold 4 of them. How many had he left?

$$\begin{array}{r} 20 \\ -4 \\ \hline ?? \end{array}$$

4 and ? make 10? 1 and ? make 2?

5. A man had 32 sheep and sold 5 of them. How many had he left?

$$\begin{array}{r} 32 \\ -5 \\ \hline ?? \end{array}$$

5 and ? make 12? 1 and ? make 3?

Note. You see that subtraction is almost the same as addition. In the fifth example, if we write in the answer, we shall have 27. The number of sheep that he sold and the number that he had left must be the same as the number that he had at first. We prove our answer by adding the lower numbers to see that they make the upper number.

$$\begin{array}{r} 32 \\ -5 \\ \hline 27 \end{array}$$

Subtract in the exercises written below.

6. $$\begin{array}{r} 34 \\ -23 \\ \hline \end{array}$$

3 and ? make 4? 2 and ? make 3?

7. $$\begin{array}{r} 32 \\ -17 \\ \hline \end{array}$$

7 and ? make 12? 2 and ? make 3?

8. $$\begin{array}{r} 72 \\ -34 \\ \hline \end{array}$$

4 and ? make 12? 4 and ? make 7?

Note to the ***Teacher.*** The additive plan of subtraction is earnestly recommended. It constantly strengthens addition, and it is much easier for the pupil to learn. If nothing is said about "borrowing," the habit of "carrying" in addition is used as a matter of course by the pupils in taking up subtraction with "borrowing." Guide the reading with the pointer, beginning at the subtrahend and reading "down and around," and in starting the reading for the next column, if there is any "carrying," hesitate a moment until the children name the right number. They will take the work very easily in a very few lessons. In the drill with objects, and where motor activity is used, some care will need to be taken by the teacher to whom the method is new, and who has to overcome her habits acquired by the older method. With splints in the hands, to show the first example given above, hold 5 splints in one hand, and then, as you repeat the problem, take 2 of them away, and showing them, say, 2 spent; then, showing the 3 left, say, 3 left. Then, putting them together as a proof, say, 2 and 3 are 5. If custom placed the line in the proper place, the association of addition and subtraction would be more easily recognized. For example,

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array} \qquad \begin{array}{r} 5 \\ \hline 2 \\ +3 \end{array}$$

In the first, or in addition, the line is between equal amounts. In the second, subtraction, it is as here written, between equal amounts. What is spent and what is left, together, equal what he had at first. As custom has established the convention of using the line lower down,

$$\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$$

we have to group *about* the line to find the whole amount shown by the minuend. To do this, we consider the line equivalent to + or "and," and begin to read at the sign −. Fix the habit of "proof" from the first.

Subtraction drill.

$$\begin{array}{r}7\\-3\\\hline ?\end{array}\quad\begin{array}{r}9\\-7\\\hline ?\end{array}\quad\begin{array}{r}5\\-2\\\hline ?\end{array}\quad\begin{array}{r}6\\-2\\\hline ?\end{array}\quad\begin{array}{r}7\\-4\\\hline ?\end{array}\quad\begin{array}{r}8\\-3\\\hline ?\end{array}\quad\begin{array}{r}9\\-4\\\hline ?\end{array}$$

$$\begin{array}{r}7\\-2\\\hline ?\end{array}\quad\begin{array}{r}6\\-3\\\hline ?\end{array}\quad\begin{array}{r}5\\-3\\\hline ?\end{array}\quad\begin{array}{r}9\\-6\\\hline ?\end{array}\quad\begin{array}{r}6\\-3\\\hline ?\end{array}\quad\begin{array}{r}9\\-3\\\hline ?\end{array}$$

$$\begin{array}{r}21\\-3\\\hline ?\end{array}\quad\begin{array}{r}31\\-3\\\hline ?\end{array}\quad\begin{array}{r}41\\-3\\\hline ?\end{array}\quad\begin{array}{r}51\\-3\\\hline ?\end{array}\quad\begin{array}{r}61\\-3\\\hline ?\end{array}\quad\begin{array}{r}71\\-3\\\hline ?\end{array}\quad\begin{array}{r}81\\-3\\\hline ?\end{array}$$

$$\begin{array}{r}91\\-3\\\hline ?\end{array}\quad\begin{array}{r}12\\-3\\\hline ?\end{array}\quad\begin{array}{r}22\\-3\\\hline ?\end{array}\quad\begin{array}{r}32\\-3\\\hline ?\end{array}\quad\begin{array}{r}42\\-3\\\hline ?\end{array}\quad\begin{array}{r}52\\-3\\\hline ?\end{array}$$

13	23	33	43	53	63	73
−4	−4	−4	−4	−4	−4	−4
?	?	?	?	?	?	?

83	93	14	24	34	44
−4	−4	−5	−5	−5	−5
?	?	?	?	?	?

Exercises in addition and subtraction.

(1)	(2)	(3)	(4)	(5)	(6)
345	452	467	432	374	782
272	432	452	436	762	432
345	679	796	976	786	435
342	435	452	452	457	796
341	103	706	980	450	320

(7)	(8)	(9)	(10)	(11)	(12)	(13)
642	721	786	932	734	820	832
−324	−252	−232	−280	−216	−423	−321

(14)	(15)	(16)	(17)	(18)	(19)
7895	9735	8976	87667	97648	98456
−3245	−4532	−5432	−34576	−57893	−672

Applications in subtraction.

1. James had 34 marbles and lost 12 of them. How many had he left?

2. Frank had 75 cents and spent 18 cents. How many had he left?

3. There are 48 children in the class, 26 of whom are boys. How many girls are there in the class?

4. A man had 75 acres of land and sold 14 acres. How many acres had he left?

5. There were 80 sheep in a field, but the farmer took out 25 of them and put them into another field. How many remained in the first field?

Write out some drill exercises in addition, and work them for yourself.

Write out some exercises in subtraction drill for yourself, and find the answers.

Write out some examples about addition.

Write out some examples about subtraction.

Note to the Teacher. Children like to make exercises and to write examples for each other and for themselves. It is wise to give them abundant opportunity to do this valuable work.

Permit them to write "number stories" on the board before session, and you will have many who will be glad to cover the board with drill exercises, etc.

MULTIPLICATION

In addition we put together different numbers of the same kind, as 3 apples and 4 apples, and find how many in all.

In subtraction we find the difference between numbers of the same kind, as 3 apples and how many apples are 4 apples?

In multiplication we use the same number or quantity a given number of times, as 3 times 4 apples are how many apples?

The sign which tells us to add is called "plus." It is written like this, +. The sign which tells us to subtract is called "minus." It is written like this, −. The sign which tells us to multiply is called the multiplication sign. It is written like this, ×. $\begin{array}{r} 4 \\ \times\ 3 \\ \hline \end{array}$ means that we are to take 4 three times, or to multiply 4 by 3.

Take 4 splints each time for 3 times and place them in a pile, and if you count them you will find 12 in the pile. Use splints or other convenient objects to "work out" the following table of 2's.

$$\begin{array}{rrrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \times\ 2 & \times\ 2 & \times\ 2 & \times\ 2 & \times\ 2 & \times\ 2 & \times\ 2 & \times\ 2 & \times\ 2 & \times\ \ 2 \\ \hline ? & ? & ? & ? & ? & ? & ? & ? & ? & ? \end{array}$$

Take 1 object each time for 2 times for the first.

Take 2 objects each time for 2 times for the second.

Take 3 objects each time for 2 times for the third.

Take 4 objects each time for 2 times for the fourth.

Take 5 objects each time for 2 times for the fifth.

In the following tables, the answers are given so that you will not have to count out so many objects. You should study these tables very carefully and learn them so that you will never forget the answers, as you will be called upon many times to use them.

3's.

$$\begin{array}{r}1\\ \times\ 3\\ \hline 3\end{array}\quad\begin{array}{r}2\\ \times\ 3\\ \hline 6\end{array}\quad\begin{array}{r}3\\ \times\ 3\\ \hline 9\end{array}\quad\begin{array}{r}4\\ \times\ 3\\ \hline 12\end{array}\quad\begin{array}{r}5\\ \times\ 3\\ \hline 15\end{array}\quad\begin{array}{r}6\\ \times\ 3\\ \hline 18\end{array}\quad\begin{array}{r}7\\ \times\ 3\\ \hline 21\end{array}\quad\begin{array}{r}8\\ \times\ 3\\ \hline 24\end{array}\quad\begin{array}{r}9\\ \times\ 3\\ \hline 27\end{array}\quad\begin{array}{r}10\\ \times\ 3\\ \hline 30\end{array}$$

4's.

$$\begin{array}{r}1\\ \times\ 4\\ \hline 4\end{array}\quad\begin{array}{r}2\\ \times\ 4\\ \hline 8\end{array}\quad\begin{array}{r}3\\ \times\ 4\\ \hline 12\end{array}\quad\begin{array}{r}4\\ \times\ 4\\ \hline 16\end{array}\quad\begin{array}{r}5\\ \times\ 4\\ \hline 20\end{array}\quad\begin{array}{r}6\\ \times\ 4\\ \hline 24\end{array}\quad\begin{array}{r}7\\ \times\ 4\\ \hline 28\end{array}\quad\begin{array}{r}8\\ \times\ 4\\ \hline 32\end{array}\quad\begin{array}{r}9\\ \times\ 4\\ \hline 36\end{array}\quad\begin{array}{r}10\\ \times\ 4\\ \hline 40\end{array}$$

5's.

$$\begin{array}{r}1\\ \times\ 5\\ \hline 5\end{array}\quad\begin{array}{r}2\\ \times\ 5\\ \hline 10\end{array}\quad\begin{array}{r}3\\ \times\ 5\\ \hline 15\end{array}\quad\begin{array}{r}4\\ \times\ 5\\ \hline 20\end{array}\quad\begin{array}{r}5\\ \times\ 5\\ \hline 25\end{array}\quad\begin{array}{r}6\\ \times\ 5\\ \hline 30\end{array}\quad\begin{array}{r}7\\ \times\ 5\\ \hline 35\end{array}\quad\begin{array}{r}8\\ \times\ 5\\ \hline 40\end{array}\quad\begin{array}{r}9\\ \times\ 5\\ \hline 45\end{array}\quad\begin{array}{r}10\\ \times\ 5\\ \hline 50\end{array}$$

Note to the Teacher. Write the tables in the natural form, as shown above, that the pupils may find their visualization of the form helpful when they use it in computation.

It is well to maintain the psychological distinction between multiplication and addition. Treat multiplication as measuring by a fixed quantity, repeated a required number of times, and addition as combining numbers of different size but of the same kind.

In "working out" multiplication tables by objects, take the *same number* of objects each time, repeating the action the required number of times, and counting the resulting accumulation.

Counting by 2's, 3's, etc., should be considered a drill in multiplication and not one in addition. (See Measuring, page 8.)

Make drill cards showing single combinations of the table, and use them freely. The sign is written between the numbers, that the pupils may read them both ways, and thus reduce the work of learning the table one half. In the tables given above, duplicates are used for the sake of rhythm. In the drill cards this is not necessary, $\times\begin{array}{c}3\\2\end{array}$ or $\times\begin{array}{c}2\\3\end{array}$ being the same as to result, and only one drill card necessary.

Applications of multiplication.

1. If 1 orange costs 4¢, what will 3 oranges cost?

Question. ? cents? (How many cents?)

Answer. 12 cents.

WORK

$$\begin{array}{r} 4 \\ \times\ 3 \\ \hline 12 \end{array}$$

2. What will 2 pounds of butter cost at 32¢ a pound?

Question. ? cents?

Answer. 64 cents.

$$\begin{array}{r} 32 \\ \times\ \ 2 \\ \hline 64 \end{array}$$

3. What will 32 apples cost at 2¢ each?

Question. ? cents?

Answer. 64 cents.

$$\begin{array}{r} 32 \\ \times\ \ 2 \\ \hline 64 \end{array}$$

To the Pupil. In the work, first set down *what is asked*, and then, if you cannot tell without doing some work at one side, make the computations at the side, and write the number thus found in the place of the question "how many." Do not mark the names in the side work, or you will have trouble with examples like the third.

4. What will 432 sheep cost at $5 each?

Question. ? dollars?

Answer. $2160.

$$\begin{array}{r} 432 \\ \times 5 \\ \hline 2160 \end{array}$$

Drill exercises with tables.

$$\begin{array}{r} 132231332113231232311223 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 321231323212312322333321 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{ccccccccc} 32 & 23 & 33 & 45 & 56 & 53 & 34 & 28 & 82 \\ \times 4 & \times 4 & \times 3 & \times 3 & \times 2 & \times 5 & \times 4 & \times 3 & \times 2 \\ \hline 128 & 92 & ? & ? & ? & ? & ? & ? & ? \end{array}$$

To the Pupil. In $\begin{array}{r} 23 \\ \times 4 \\ \hline \end{array}$ we have for the first part of our answer 12, but as there is only *room* to write the 2, we carry the 1 over to the next place, and when we have 8 in that place, add the 1, and so have 9 in one place and 2 in the other, or 92. Think that you are filling boxes, standing side by side, with strawberries, and when one is full, you put the rest of the berries in the box next at your left, and keep on doing this as fast as you fill the boxes.

See if you can do this hard exercise.

$$\begin{array}{r} 43675 \\ \times 5 \\ \hline ?????? \end{array}$$

The answer is 218375. Can you fill the boxes correctly and get that answer?

To the Teacher. It is not necessary to enter upon any elaborate explanation of the process of multiplying at this stage of the progress of the pupils. A little later, when we consider place value of figures, and take up reduction of denominate units, this whole matter can be explained in detail.

The troublesome question of marking the numbers used in the computation and avoiding abuse of arithmetical principles, is best adjusted by *first* deciding the terms of the answer, then making the computations mentally as far as possible. When written computation is necessary, it should be done in the abstract form.

In the fourth example given above,

$$\begin{array}{r} \$5 \\ \underline{432} \\ \$2160 \end{array}$$

is often used.

$$\begin{array}{r} 432 \text{ sheep} \\ \underline{\$5} \\ \$2160 \end{array}$$

is too often seen on the papers of children.

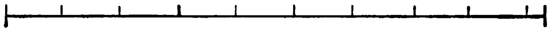
MEASURING

If you were to tell some one that the top of your desk is 4 times the length of your pencil, he would not know how long your desk is unless he saw the length of your pencil.

If you were to tell him that your desk is 9 times as long as a 2-inch measure, he would know its length, as he knows the length of the measure which you used.

Ask your teacher to make you a measure 2 inches long (from a match) and then use this to measure some of the things suggested here:

How long is this book? How wide is this book? How long is this chalk line which I have made on the board?



Answer. It is a little more than 9 times as long as my 2-inch measure, or it is a little more than 18 inches long.

When older people measure, they keep the total 2, 4, 6, 8, 10, 12, 14, 16, 18 as they repeat the use of their measure. Can you do this with your 2-inch measure?

Can you make a 4-inch measure from a small piece of wood, using your 2-inch measure to tell you how long to make it?

Make measures of different lengths, and measure many things.

A measure 12 inches long is called a *foot* measure.

A measure 3 feet long is called a *yard* measure.

Get a stick long enough and make a yard measure, then use it to measure long objects or long distances.

Could you measure water or milk with any of these measures?

To measure water, milk, and other liquids, we use measures called pints, quarts, and gallons. Have you ever seen these measures?

If you can get some of these measures, you may use them in measuring water, and in testing the following table:

One quart is 2 pints.
One gallon is 4 quarts.
One gallon is 8 pints.

To the Teacher. The work in measuring, using measures made by the pupils, and not marked off into confusing subdivisions, but used for entire length at first, is a matter of considerable interest to them, and also one of great importance in its foundation for the multiplication table, etc. In early times the farmers cut a rod or pole $16\frac{1}{2}$ feet long and used it to measure out land, from which we get our rod, perch, or pole, even now used in some sections for land measurement. If the actual measures for liquid and dry measures cannot be obtained for use, have them fix the comparative size by pictures and applications. Do not permit them to get the too common impression that a dry pint or quart is the same size as a liquid pint or quart. Do not ask them to find surface measure until you can give them the proper unit for such measure. If you think it necessary to introduce surface measure at all, give the pupils a *square* unit of measure and let them measure the surfaces by repetition of this measure.

They can make diagrams showing the work, and in the written work they should use either, 4 square inches × 3 = 12 square inches, or, size 4 inches by 3 inches, surface ? square inches, and write in the proper numbers for the question mark. Do not use the dangerous expression, "multiply the length by the breadth."

DIVISION

If I should place a pile of 16 pennies on your desk, and ask you how many apples you could buy with them if each apple cost you 2 cents, you would take 2 cents out of the pile as many times as possible, until it was all gone, and then tell me how many 2's you had taken. You find the answer to the question, 18 is how many 2's? You have learned in multiplication that 9 2's are 18. Can you answer the other question, — 18 is how many 2's? — without taking objects and counting it out?

Division is the reverse of multiplication.

In division we find how many times one number or quantity contains another.

1. At 6 cents each, how many pencils can I buy for 24 cents?

Question. 24 is how many 6's?

Answer. 24 is 4 6's.

Work	Proof
$\overline{4}$	$\times\ 4$
$6\overline{)24}$	$6\overline{)24}$

In reading the work, we read 24 is 4 6's. In reading the proof, we read 4 6's are 24. Do you see how close is the relation between division and multiplication?

2. How many yards in 21 feet? *Ans.* 7.

Each yard is 3 feet.

Question. 21 feet is how many times 3 feet?

Answer. 21 feet is 7 times 3 feet.

WORK

Question form: $\begin{array}{r} ? \\ 3\overline{)21} \end{array}$ Read 21 is how many 3's?

Answer form: $\begin{array}{r} 7 \\ 3\overline{)21} \end{array}$ Read 21 is 7 3's. *Ans.* 7.

3. Bought 8 apples for 24 cents. How much did I pay for each?

Question. 24 cents for 8 apples, how many cents for 1 apple?

Answer. 3 cents.

WORK

$\begin{array}{r} 3 \\ 8\overline{)24} \end{array}$ *Ans.* 3 cents.

4. Write the answers to the following:

$$\begin{array}{r} ? \\ 7\overline{)28} \end{array} \quad \begin{array}{r} ? \\ 6\overline{)18} \end{array} \quad \begin{array}{r} ? \\ 5\overline{)20} \end{array} \quad \begin{array}{r} ? \\ 4\overline{)16} \end{array} \quad \begin{array}{r} ? \\ 3\overline{)27} \end{array} \quad \begin{array}{r} ? \\ 6\overline{)24} \end{array}$$

$$\begin{array}{r} ? \\ 9\overline{)27} \end{array} \quad \begin{array}{r} ? \\ 4\overline{)32} \end{array} \quad \begin{array}{r} ? \\ 3\overline{)15} \end{array} \quad \begin{array}{r} ? \\ 2\overline{)14} \end{array} \quad \begin{array}{r} ? \\ 7\overline{)49} \end{array}$$

In reading the above, read them, 28 is how many 7's? 18 is how many 6's? etc.

Division where we have something remaining.

1. $\begin{array}{r}3\frac{1}{2}\\2\overline{)7}\end{array}$ We write what remains over the divisor to show that that part has not been divided.

2. $\begin{array}{r}36763\\2\overline{)73526}\end{array}$ This is hard, so you must study it carefully. We work as follows: 7 is 3 2's and 1; then use the 1 before the 3, as there is where it belongs, it having been in that "box"; and read 13 is 6 2's and 1; 15 is 7 2's and 1; 12 is 6 2's and none; 6 is 3 2's.

3. Try this one, which is yet harder. $\begin{array}{r}14368\\3\overline{)43104}\end{array}$

This is the way we think it out as we work: 4 is 1 3 and 1; 13 is 4 3's and 1; 11 is 3 3's and 2; 20 is 6 3's and 2; 24 is 8 3's. 14368 *Ans.* To prove our answer, we multiply it by 3 to see if it gives the number with which we started. Can you do this without writing the numbers in a different form? You should learn to do so, and should prove your answer in each exercise.

4. Divide 67 by 3. $\begin{array}{r}?\\3\overline{)67}\end{array}$ $22\frac{1}{3}$ *Ans.*

5. Divide 678 by 4. 6. Divide 3452 by 3.

1. $\begin{array}{r}3\\2\overline{)6}\end{array}$ $\begin{array}{r}3\frac{1}{2}\\2\overline{)7}\end{array}$

Where there is a remainder after dividing as far as possible, we write the remaining or undivided part over the divisor as a part of the quotient.

2. $\begin{array}{r}2\\3\overline{)6}\end{array}$ $\begin{array}{r}2\frac{1}{3}\\3\overline{)7}\end{array}$ $\begin{array}{r}2\frac{2}{3}\\3\overline{)8}\end{array}$ $\begin{array}{r}3\\3\overline{)9}\end{array}$ $\begin{array}{r}3\frac{1}{3}\\3\overline{)10}\end{array}$ $\begin{array}{r}3\frac{2}{3}\\3\overline{)11}\end{array}$ $\begin{array}{r}4\\3\overline{)12}\end{array}$ $\begin{array}{r}4\frac{1}{3}\\3\overline{)13}\end{array}$

3. Divide by 4, that is, find how many 4's in: 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

4. How many 5's in: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 ?

5. How many 6's in: 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 ?

6. How many 7's in: 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 ?

7. How many 8's in: 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45 ?

8. How many 9's in: 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72?

9. How many 3's in 43104?

$$\begin{array}{r} 14368 \\ 3\overline{)43104} \end{array}$$

Instead of writing the remainder, after dividing the first figure by 3, we use it in its proper place, which is before the next figure of the dividend. This remainder is 1, and used before the next figure it gives us 13 for our next partial dividend, 13 is 4 3's and 1, and as we use this remainder of 1 our next partial dividend is 11. 11 is 3 3's and 2; 20 is 6 3's and 2; 24 is 8 3's.

This division is just the same as that shown in the exercises above, except that we use the remainder and continue the division, instead of writing it in the quotient.

We prove our work by multiplying the quotient by the divisor to see if it gives the dividend. You can do this without rewriting the numbers, after a little practice, and should prove each exercise in division in that manner.

10. Divide 5349 by 3.

11. Divide 8744 by 4.

12. Divide 9144 by 6.

Drill form for concert readings to fix habit, and to strengthen tables.

$$3\overline{)54321023456784329537624586734253}$$

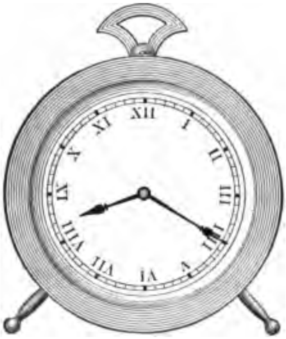
Class reads: 5 is 1 3 and 2; 24 is 8 3's; 3 is 1 3; 2 is no 3; 21 is 7 3's; 0 is no 3; 2 is no 3; 23 is 7 3's and 2, etc.

Note to the Teacher. The written form where the quotient is used over the dividend is important, in that no great break is made when "long" division is introduced, and none when division of decimals is introduced. It also aids the children in the test of the answer by multiplying the quotient by the divisor; as the child will not need to write these numbers at the side for proof in "short" division, but can easily be guided to multiplication in their present positions.

The idea of beginning with the dividend and seeking the answer to the question "how many of the required units are here contained?" instead of using the older and probably more familiar form of "3 in 6 how many times?" has been outlined here only after careful test of its efficiency in the classroom with many classes. It is found that this form is more natural to the child, that it lends itself much more easily to motor activity in taking out of a pile a set number, as many times as the pile will permit. It has been found to be very important in overcoming the trouble about "forgetting if not written in" the remainders in "short" division, as in the form here used this remainder is at once applied in the form which is natural to the pupil. In "long" division, the chief trouble clusters around the question, "How many times?" and here again the form suggested has proved its value by actual test in classroom use.

If we begin with the dividend instead of the divisor in all of our work in division, we may have our own prejudices, based upon long habit, to overcome; but we will find no difficulty with the children taking the topic for its first consideration.

Time by the clock and introduction to Roman notation.



I	II	III	IIII	V	VI	VII	VIII	IX	X	XI	XII
1	2	3	4	5	6	7	8	9	10	11	12

The short hand over the dial of the watch or clock tells the hour, the long hand tells the minute.

Each space between the figures on the dial is 5 minute spaces in length.

When the long hand is at I, it is 5 minutes past the hour shown by the short hand; when at II, it is 10 minutes past the hour shown by the short hand. When the long hand is at III, it is 15 minutes, or one quarter of an hour, after the hour which the short hand has passed. When the long hand is at VI, it is 30 minutes, or half past the hour which the short hand has passed. When the long hand is at IX, it is one quarter of the next hour which the short hand will reach.

If you do not know how to tell time by the clock, you should learn to do so at once.

A good way to learn is to make a picture of the face of the clock, and then with two pieces of paper shaped like the hands of the clock, and pinned to the center as they are, you can move the hands into different positions, and tell the time shown in each position to some one who is older, and who will help you if you have trouble in telling the time shown in any position of the hands.

The letters I, V, and X used on the dial of clocks have been used many years, and because they were once used by the people called Romans, this way of writing numbers is called the Roman Notation.

For numbers larger than 12, they use other letters:

L for 50, C for 100, D for 500, M for 1000.

You should know how to read these numbers, although you will never use them much. Can you read the numbers of the chapters in the Bible? Open the Bible at the Book of Psalms, and see if you can read the numbers written there.

XIII	XIV	XV	XVI	XVII	XVIII
13	14	15	16	17	18

XIX	XX	XXX	XL	L	LX	LXX
19	20	30	40	50	60	70

LXXX	XC	C	D	M
80	90	100	500	1000

INTRODUCTION TO FRACTIONAL PARTS

We often hear people speak of half an apple, half a dollar, quarter of an hour, quarter of a dollar, etc.

Let us see what these terms mean.

One half means one of two *equal* parts. Written $\frac{1}{2}$.

One third means one of three equal parts. Written $\frac{1}{3}$.

One fourth means one of four equal parts. Written $\frac{1}{4}$.

One fifth means one of five equal parts. Written $\frac{1}{5}$.

One sixth means one of six equal parts. Written $\frac{1}{6}$.

If we divide an apple into two parts, we must not call the parts halves, unless we have divided the apple into two *equal* parts. We call the parts that are not equal, "pieces." Could you cut an apple so as to give me $\frac{1}{2}$ of it? Could you cut it so as to give me a piece of it? If you gave me $\frac{1}{2}$ of it, and kept the remainder, which of us would have the larger part of the apple?

Take this paper and fold and crease it, or draw a pencil mark across it, so that you can show me one half of it.

Can you show me what we mean by $\frac{1}{3}$? By $\frac{1}{4}$? By $\frac{1}{5}$?

Keep the picture, or the paper which you have creased, before you and see if you can answer these questions:

If one pound of sugar cost 6 ¢, what will $\frac{1}{2}$ pound of sugar cost?

What is $\frac{1}{2}$ of 10?
What is $\frac{1}{2}$ of 8?
What is $\frac{1}{2}$ of 12?
What is $\frac{1}{3}$ of 6?
What is $\frac{1}{3}$ of 12?
What is $\frac{1}{3}$ of 18?

Which is the larger, $\frac{1}{2}$ of an apple or $\frac{1}{3}$ of an apple?

If you cut a pie into 4 equal parts, what may you call one of those parts?

To the Teacher. In all work with fractions, keep the whole unit prominently before the pupil. Do not break a stick into two parts and call the parts halves. They are wholes as soon as fully separated, and the first unit destroyed. Use diagrams and the folded paper freely. Do not use a standard type of diagram, but vary the form, always suggesting completeness. A single line is not as good as a rectangle, as the line does not suggest completeness as thoroughly.

Have the pupils make sketches to show the value of fractional expressions, and to show application of fractional forms in problems. It is more important to train the pupils to make sketches to show their thought, than to entertain them by showing them colored diagrams or "pie-pictures" in books.

ELEMENTARY ARITHMETIC

CHAPTER I

READING AND WRITING NUMBERS

1. Introduction. You have learned to read and write numbers written with three figures, such as 125, 754, 345, etc. It is easy to learn to read larger numbers, if we divide them into groups of three figures each, beginning at the *right* and pointing off toward the left. These groups or periods of figures have names which we must use in reading them. We have learned to read in the first group; the next larger group is called *thousands*. To read 435678, we point it off into groups, — 435,678, — and then read 435 thousand 678. 56,432 is read 56 thousand 432. 75,305 is read 75 thousand 305.

2. Read the following numbers:

1. 35,987. 453,213. 536,309. 54,375.
2. 7,456. 25,659. 150,237. 203,105.
3. 350,275. 540,348. 675,009. 590,007.
4. 4,007. 14,006. 25,005. 305,127.
5. 126. 368,215. 125,003. 175,000.

3. Point off into periods of three figures each, and then read:

1. 345065. 257431. 389054. 450932.
2. 10575. 3285. 204357. 50005.
3. 357940. 243617. 35218. 20504.
4. 100100. 300300. 500500. 300001.
5. 200405. 400503. 70007. 80005.

4. The next period of larger numbers, that is the group next larger than thousands, is called *millions.* We read numbers large enough to use this group, just as we have read the smaller numbers, and then give it the name of the group. 145,000,000 is read 145 millions. 145,136,234 is read 145 millions 136 thousand 234.

5. Point off and read the following numbers:

1. 3456905. 4567809. 534675120. 4536783.
2. 34567207. 230876. 35765321. 231007135.
3. 60800723. 4006004. 15065730. 125670802.
4. 8000008. 4000400. 3000305. 100000210.
5. 3540052. 2109874. 3210542. 3567098.

6. Write in figures:

1. Fifteen thousand, four hundred twenty-one.
2. Forty-eight thousand, fifty-five. Twenty thousand, one hundred eighty-five.

3. Two hundred eighteen thousand, five hundred sixty-eight.

4. Twenty thousand, two hundred five. Thirty thousand, one hundred ten. Five thousand five. Two hundred seven.

5. One million, three hundred thousand, two hundred sixty-five.

6. Twenty million, three hundred thousand, one hundred eighteen.

7. Ninety-five thousand, six hundred twenty-four. Two thousand two.

8. Five thousand, fifty. Three thousand, twenty. One thousand, five. Twenty thousand, four.

9. Four hundred sixteen thousand, two hundred twenty. One hundred forty-six million, three hundred forty-five thousand, one hundred ninety-seven.

10. Two hundred forty thousand, two hundred forty. Three hundred six million, three hundred six thousand, three hundred six.

7. Reading numbers of dollars and cents.

The sign $ is used for dollars. The mark used to separate the dollars from the cents is the .

$4.25 is read four dollars, twenty-five cents.

$5.18 is read five dollars, eighteen cents.

$125.10 is read one hundred twenty-five dollars, ten cents.

8. Read the following:

(1)	(2)	(3)	(4)	(5)
$12.45.	$56.75.	$5.08.	$125.04.	$250.05.
$14.50.	$5.26.	$15.06.	$123.07.	$245.04.
$124.37.	$15.14.	$15.07.	$150.04.	$248.04.
$145.18.	$246.20.	$15.03.	$156.05.	$1245.03.
$102.25.	$1245.20.	$4.02.	$130.09.	$1542.05.

9. Write the following, using figures:

1. Twelve dollars, twenty-five cents.

2. Two hundred dollars, thirty-six cents.

3. Five dollars, sixteen cents. Four dollars, fifty cents.

4. Five dollars, five cents ($5.05). Six dollars, five cents.

5. Seven dollars, six cents. Eight dollars, four cents.

6. Twenty dollars, five cents. Thirty dollars, four cents.

7. One hundred dollars, six cents. Eighty-five dollars, five cents.

8. Five cents. Eight cents. Six cents. Three cents. Two cents.

9. Two dollars, four cents. Eight cents. Twenty-five cents.

10. Thirty cents. Fifty cents. Seventy-five cents.

ADDITION

10. 1. Add the following numbers:

2345; 4578; 3695; 2648.

For convenience, we usually write the numbers to be added in columns.

(2)	(3)	(4)	(5)	(6)
24532	3462	54879	237859	789678
54243	7465	84765	987346	986789
13541	6452	87421	841236	986578
35214	6423	76419	467852	768496

(7)	(8)	(9)
34671	784569	879456
432	8756	98745
1461589	51	986
345	9876454	1437689
54134	117	349

11. In writing the following, keep the right-hand column straight up and down. Arrange the following in columns and then add them:

1. 63,481; 376; 1,345,976; 233; 1,456,245.
2. 4578; 133; 45,267; 348,960; 321.
3. 87609; 347; 986521; 3456; 121.
4. 78,963; 999; 908,742; 187,645; 131.
5. 876,909; 887; 679,084; 89,763; 99.
6. 367; 97,608; 999,786; 887; 4,501,768.

7. 876,490; 1678; 998; 678,320; 89,911.
8. 67,834; 7761; 98,876; 999,998.

Addition of dollars and cents.

12. We may add dollars and cents just as we add simple numbers, if we are careful to place them in columns so that the periods separating the dollars from the cents stand directly under each other, and if we write the period in the answer directly under those in numbers added.

Add the following:

(1)	(2)	(3)	(4)
$15.25	$16.25	$25.45	$25.13
25.16	23.15	20.25	8.10
13.24	12.35	8.37	120.24
25.10	18.21	16.12	23.14
$78.75	$.	$.	$.

13. Business men use a vertical line to divide the dollars from the cents, in arranging columns for addition. In this way:—

$$\begin{array}{r|l} \$23 & 45 \\ 8 & 16 \\ 15 & 20 \\ 3 & 25 \\ \hline \$50 & 06 \quad \textit{Ans.} \end{array}$$

1. Add: $67.89; $75.05; $43.60; $7.53; $12.45; $10.65.

2. $150.65; $195.65; $12.57; $65.15; $175.10; $185.14.

3. $150.25; $65.; $3.50; $27.04; $.45; $50.90; $10.15; $.25.

4. $160.25; $75.; $1.45; $28.50; $.75; $7.50; $8.64; $15.20.

5. $320.25; $1.50; $78.65; $89.75; $125.36; $45.29; $75.

6. $675.10; $780.75; $750.; $980.35; $125.18; $145.26.

7. $1250.75; $3456.70; $6543.26; $4520.80; $8674.30.

8. $2500.; $150.; $10.50; $250.65; $675.14; $750.

9. $750.45; $89.75; $65.45; $5.10; $25.75; $1250.15.

10. $250.79; $675.10; $875.46; $575.40; $5.40; $.80.

14. Applications.

1. James weighs 65 pounds, and Fred weighs 75 pounds. How much do they both weigh?

2. Frank had $5 and earned $2.50 more. How much had he then?

3. I buy at the store some bread for $.12, some tea for $.45, some sugar for $.75, and some butter for $.90. What did they all cost?

4. A farmer sold some cows for $ 150, a horse for $ 175, and some hay for $ 35. What did he receive for all?

5. A boy sold 20 newspapers on Monday, 25 on Tuesday, 30 on Wednesday, 32 on Thursday, 28 on Friday, and 45 on Saturday. How many papers did he sell in the whole week?

6. A farmer sells some potatoes for $10.50, some apples for $9, some oats for $35, and some butter for $25.50. How much did he receive for all?

7. A boy paid 3 cents for some marbles, 15 cents for some candy, 12 cents for a book, and 50 cents for a knife. How much money did he spend?

8. A man spent $10 for some straw, $15 for some hay, and $5.50 for some grain. How much money did he spend?

9. James gathered 45 eggs on Tuesday, 38 on Wednesday, 42 on Thursday, and 37 on Friday. How many did he have in all?

10. William is 15 years old, Mary is 5 years older than William, and Frank is 6 years older than Mary. How old is Frank?

11. A farmer digs 250 bushels of potatoes in one field, 54 bushels in another, and 75 bushels in another. How many bushels did he dig in all?

SUBTRACTION

15. Have you ever noticed how business men make change? They add to the smaller number until the sum is the larger number.

If you buy 15 cents' worth of candy, and give the clerk 25 cents; when he makes the change he will hand you the candy, saying "15," and then the change, 10, which he *adds* to the 15 to make 25 cents in value which you gave him.

You gave him,	He gives you,
25 cents.	15 cents (value in candy)
	10 cents in change.
	25 cents in change and candy.

This might be written out as follows:

$$\begin{array}{ll} \underline{25\text{¢}} & (\text{¢ is the business sign for cents}) \\ 15\text{¢} & (\text{candy}) \\ ?\text{¢} & (\text{change}) \end{array}$$

In this form we use the line between equal values, and see at once that we should receive enough change to add to the 15¢ to make the value the same as the 25¢.

We can, however, do the work in this business way, and use the old-fashioned written form, which is as follows:

$$\begin{array}{r} 25\text{¢} \\ \underline{-\,15} \\ ?\text{¢} \end{array}$$

In this form, we begin with 15 and think, 15 and how many are 25? We begin at the number to be subtracted and read "down and around" to the larger number. We read the question, — 15¢ and how many ¢ make 25¢? We read as we fill in the answer, — 15¢ and 10¢ are 25¢.

$$\begin{array}{r} 65 \\ -23 \\ \hline \end{array}$$

Notice how we read this exercise. Begin with the 3, reading down. Calling the line "and" we read, "3 and what are 5?" for our question, and write in 2 for the answer; then "2 and what are 6?" and write in 4 for the answer. We have thus found the number which must be added to 23 to make it equal to 65; that is, we have found the *difference* between 23 and 65, which is 42.

$$\begin{array}{r} 396 \\ -354 \\ \hline \end{array}$$

Read it, 4 and ? (2) are 6; 5 and ? (4) are 9; 3 and ? (5) are 8. Begin at the number which has the sign of subtraction, and read down and around to the upper number, writing in the number below the line for the answer.

$$\begin{array}{r} 765 \\ -342 \\ \hline \end{array}$$

2 and ? are 5; 4 and ? are 6; 3 and ? are 7.

$$\begin{array}{r} 863 \\ -335 \\ \hline \end{array}$$

5 and ? are 13; 4 and ? are 6; 3 and ? are 8.

$$\begin{array}{r} 734 \\ -146 \\ \hline \end{array}$$

6 and ? are 14; 5 and ? are 13; 2 and ? are 7.

950 – 375	5 and ? are 10; 8 and ? are 15; 4 and ? are 9.
4376 – 2189	9 and ? are 16; 9 and ? are 17; 2 and ? are 3; 2 and ? are 4.

Note to the Teacher. If additive-subtraction is new to the pupils, and they have learned to subtract in the old-fashioned way, you can easily teach them the better way in four or five lessons.

Guide the reading on board exercises by the movements of the pointer, and have them read down and around from the subtrahend, through the remainder to the minuend, writing in the remainder as they read, step by step. When carrying is necessary, depend upon the habit of adding which they have learned in addition, and do not discuss the reduction of higher unit or borrowing in subtraction which took so much time and was the source of so much error in the old-fashioned way. If they hesitate slightly at first, hold the pointer a moment on the new subtrahend figure until they note its change by carrying, and then proceed. For first lessons, or until the "swing" of the work gets started, use exercises with no carrying, then with carrying for every figure, then with mixed carrying and non-carrying exercises. Experience has shown that the pupils in second year work will take this plan in three to five lessons if they are not hindered by the teacher. After they obtain the answers, teach them to check them by adding *up*, to see if the sum equals the minuend.

16. Find the remainders:

(1)	(2)	(3)	(4)	(5)
6746	5698	45496	45765	34219
– 3523	– 4365	– 32164	– 32423	– 13108

(6)	(7)	(8)	(9)	(10)
$45.24	$52.16	$65.75	$48.45	$548.35
– 23.10	– 30.15	– 42.50	– 16.25	– 435.20
$	$	$	$	$

(11)	(12)	(13)	(14)	(15)
3456	5432	3476	5432	4635
– 2137	– 4116	– 1358	– 4314	– 2417
(16)	(17)	(18)	(19)	(20)
$62.35	$56.12	$545.21	$321.15	$452.18
– 42.18	– 43.05	– 432.03	– 210.06	– 231.09
(21)	(22)	(23)	(24)	(25)
$155.05	$235.08	$256.04	$275.03	$264.07
– 132.12	– 123.15	– 132.15	– 143.15	– 131.25
(26)	(27)	(28)	(29)	(30)
14536	56374	23654	356235	25219
– 7213	– 26215	– 11875	– 163129	– 17042

17. Applications.

1. A boy had 50 cents and spent 25 cents. How much had he left?

2. A man had 50 sheep and sold 25 sheep. How many had he left?

3. A farmer had 50 bushels of oats and sold 25 bushels. How many bushels had he left?

4. There were 50 oranges in a box, and 25 were taken out. How many oranges were left in the box?

Note. You see that we can subtract any numbers, if the numbers in the same exercise are alike.

Could you subtract 50 cents from 50 oranges?

5. There were 175 acres in a farm, and 90 were sold. How many acres remained?

6. A man had $750 and spent $135.50. How much had he left?

7. A boy bought 230 marbles, and sold 75 to his friends. How many marbles had he left?

8. There are 196 pounds in a barrel of flour. How many pounds would remain, after 75 pounds had been taken out?

9. There were 750 pupils in a school, but 135 left. How many remained in the school?

10. A train has to run 350 miles from one place to the other. How much farther will it have to go, after it has run 275 miles?

ADDITION AND SUBTRACTION

18. Add:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
3456	3251	6741	4532	6798	7624	897
325	769	537	462	976	986	134
4532	4632	8953	657	978	3654	89
1234	5432	3567	2576	7853	4673	875
8753	987	764	875	752	347	908
503	2135	5025	3567	2137	2468	97

The sign + means add. It is called the *plus* sign.

19. Find the answers. The answer is called the *sum*.

(1)	(2)	(3)	(4)	(5)
3542	5793	4563	460	7840
+ 2145	+ 6432	+ 7459	+ 470	+ 7654

(6)	(7)	(8)	(9)
678	4679	3456	327
+ 680	+ 6784	+ 4532	+ 698

The sign – means subtract. It is called the *minus* sign.

The answer in subtraction is called the *remainder.*

We use the signs + and – that we may tell what to do without writing the full words, add or subtract.

20. Find the answers. The sign will tell you whether to add or to subtract.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
4562	7542	897	543	5426	5764	4357
– 3241	+ 4537	– 542	– 327	+ 789	– 4323	+ 4675

Note. In arranging numbers for subtraction, always write the larger number at the top of the column.

(8)	(9)	(10)	(11)
\$ 45.25	\$ 75.10	\$ 9.05	\$ 17.30
– \$ 12.10	+ \$ 20.05	– \$ 2.65	– \$ 10.25

(12)	(13)	(14)
\$ 60.45	\$ 37.20	\$ 12.15
+ \$ 18.20	– \$ 9.15	– \$ 4.10

21. What is the sum of \$ 15.25 and \$ 10.50?
Arrange the written work as follows:

\$ 15.25
\$ 10.50
\$ 25.75 *Ans.*

1. What is the sum of $854.35 and $65.10?

2. What is the sum of $5.25; $12.50; $6.75; $45.24?

3. Bought some candy for 15 cents; some peanuts for 10 cents; a pencil for 6 cents. What did they all cost?

4. Bought some candy for 15 cents, and gave the man a 25-cent piece. How much change should he give to me?

Written form for work:

$$\begin{array}{r} 25¢ \\ -\,15¢ \\ \hline 10¢ \end{array} \; \textit{Ans.}$$

5. A man had $50, and spent $15.50 for an overcoat. How much money had he left?

$$\begin{array}{r} \$\,50.00 \\ -\,\$\,15.50 \\ \hline \$\,34.50 \end{array} \; \textit{Ans.}$$

22. Read each example carefully, and see if you should add or subtract the numbers to answer the question.

1. If I earn $25 and spend $12.50, how much money have I saved?

2. A boy is 14 years old now. How old will he be 9 years from now?

3. A boy buys papers for 40¢, and sells them for 75¢. What did he gain?

4. A man buys cows at $40 each, and sells them at $48 apiece. How much does he make on each cow?

5. Find the sum of $456.28 and $56.79.

6. Find the remainder when $25 is taken from $100.

7. A boy had 25 marbles, and bought 15 more. How many did he have then? If he should lose 8 marbles, how many would he have left?

8. There are 16 ounces in a pound. How many ounces will remain of a pound of tea, after 7 ounces have been used?

9. Bought a pair of skates for $2.75, and gave the clerk $5.00. How much change should he return?

10. By selling a book for 75 cents, I lost 10 cents. What did the book cost me?

23. In some of these examples, you will have to add some of the numbers to find the sum of all, and then subtract another number from that sum to answer the question.

1. I had 45¢, and Frank gave me 5¢ more. I then bought some candy for 15¢. How much did I have left?

2. A farmer has 75 bushels of oats in one bin and 60 bushels in another bin. If he sells 100 bushels, how many bushels will he have left?

3. From a can of milk which held 40 quarts, 8 quarts leaked out. The man then poured 5

quarts back into the can. How many quarts of milk were in the can at last?

4. From the sum of 2456, 453, 580, 564, and 45, take 1750.

5. Bought gloves for $1.25; hat for $2.50; shoes for $3.50; and gave the clerk $10. How much change should he return?

6. Bought hay for $25; grain for $15; and a wagon for $45; and gave the man $100. How much should he give me for change?

7. A lady takes to the store butter that is worth $2.50, and then buys tea, coffee, and sugar for $1.30, and takes the rest in money. How much money does she receive?

8. James weighs 54 pounds; Frank weighs 48 pounds. Their father weighs 175 pounds. How much more does the father weigh than both of the boys?

9. A wagon loaded with hay weighs 4500 pounds. If the wagon alone when empty weighs 1250 pounds, how much does the hay weigh which was on the wagon?

10. A box filled with tools weighs 98 pounds. The empty box weighs 12 pounds. How many pounds do the tools weigh?

11. From one thousand dollars subtract $375.18.

COST AND CHANGE DRILLS

24. We can find in either of two ways the change which we should receive when we hand the clerk more than enough money to pay for the things which we have bought. We can find out what they all cost by addition, and then subtract that sum from the sum of money which we gave the clerk, and find the change to be returned. Business men have learned a quick way to do this, which saves writing the work out twice, and also saves many mistakes. Let us try to learn the way to do it quickly and surely.

1. If I buy things at the store that cost 15 cents, and give the clerk 25 cents, how much change should he return?

Operation
15¢
___ change
25¢ cash

Add down, beginning at the top, and saying, 5 and 0 are 5, writing in the 0 for change, and then reading the next column, 1 and 1 are 2, writing in the 1 as change. We have written in 10 for change, which is the right answer.

2. If I buy things at the store which cost $ 1.25, and give the clerk $ 2, how much change should he return?

Written down it is:
$ 1.25
_____ change
$ 2.00 cash

Read down and add, saying, 5 and 5 are 10, then carry, as we have learned to do in addition, and read down the next column, saying, 3 and 7 are 10; carrying again, read down the next column, saying, 2 and 0 are 2. We have written in the change as we read down, and have written in $ .75, which is correct.

3. I buy candy for 25¢; peanuts for 10¢; and a pencil for 3¢. I give the clerk 50¢. How much change should I receive?

Written down it is:
$ .25
.10
.03
_____ change
$ .50

We begin at the top of the last column, and read down, saying, 5, 8, and 2 are 10; writing in the 2 for change; then at the top of the next column, reading down, saying, 3, 4, and 1 are 5, writing in the 1 for change, and we find that we should receive $ .12 change.

4. Bought flour for $1.35; meal for $.70; tea for $.45; sugar for $.55; and matches for $.12. How much change should I receive from $5?

Written out it is:

$1.35
.70
.45
.55
.12
_____ change
$5.00

See if you can read it without asking the teacher to help, and fill in the change as you read.

5. I bought some things at the store which cost me $1.50, $25, $1.10, $3.50, and $.25. If I give the clerk $10, how much change should I receive?

COST AND CHANGE DRILL AND ADDITION

(6)	(7)	(8)
Cost $3.45	Cost $6.25	Cost $8.90
3.12	.40	2.17
.80	4.37	1.50
6.15	2.30	7.20
_____ change	_____ change	_____ change
$15.00	$20.00	$20.00

(9)	(10)
Cost $7.15	Cost $150.00
3.16	75.15
4.20	8.25
_____ change	_____ change
$15.00	$300.00

11. Bought at the store, shoes for $3.25; rubbers for $.90; and some shoe polish for $.25. Gave the clerk $5. How much change?

12. Bought at the store a book for $.75; some paper for $.36; some ink for $.08; and some pens for $.06. Gave the man a two-dollar bill. How much change should I receive?

(13)	(14)	(15)	(16)
245689	34528	$567.25	$356.25
46782	4532	75.35	536.10
56721	6374	345.80	85.25
8965	769	84.30	769.10
67321	7653	547.20	85.30
8547	3469	85.30	7.85
Sum	Sum	Sum	Sum

17. Add the following numbers: 5463; 4536; 5784; 789; 5645.

18. Find the sum of 4568; 7569; 3469; 4653; 5632; 2467.

19. A man had 35 sheep in one field, 45 in another, 50 in another, and 125 in another. How many sheep had he in all?

20. A wagon weighs 1500 pounds. If I load it with 2500 pounds of hay, what will be the entire weight of the wagon and the hay?

21. A man stands on the scales to be weighed, and finds that he weighs 165 pounds. He then weighs his son, and finds that the boy weighs 67 pounds. If he should take the boy in his arms and then stand upon the scales, how much weight would be shown?

22. A boy paid 25 cents for a book, and 15 cents more for a knife than he paid for the book. How much did he pay for the book and the knife?

23. After selling 350 sheep, a farmer has 455 sheep left. How many had he at first?

24. If I buy goods at the store that amount to $ 4.68, and give the man $ 5, how much change should I receive?

25. If I buy goods at the store that cost $ 17.25, and give the man $ 20, how much change should I receive?

26. A farmer takes potatoes, grain, and butter to the store, and sells them for $ 35.75. If he takes sugar, flour, and tea, for part of the payment, and money for the rest, and the sugar, flour, and tea cost him $ 12.50, how much money should he receive?

27. A farmer, after using 38 bushels of oats from a bin, finds that he has 135 bushels left in the bin. How many bushels were there in the bin at first?

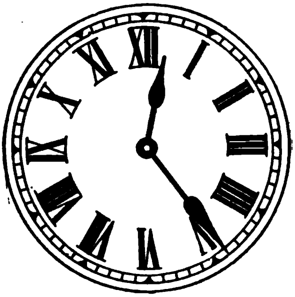
28. A man has $ 2500 in the bank, and he puts $ 350 and $ 75 more in the bank. How much has he in all in the bank?

ROMAN NUMERALS

25. Although this notation is little used now, we need to learn to read small numbers when written as the Romans wrote them long ago, as they are still used on the face of clocks and in some other places. Letters are used instead of figures.

The numbers from one to twelve on the face of the clock are:

I II III IV V VI VII VIII IX X XI XII
for
1 2 3 4 5 6 7 8 9 10 11 12.



We notice that they used I for one, V for 5, and X for ten.

They also used L for 50, C for 100, D for 500, M for 1000.

Roman,	XIII	XIV	XV	XVI	XVII	XVIII
Common,	13	14	15	16	17	18

Roman,	XIX	XX	XXI	XXII	XXIII
Common,	19	20	21	22	23

Roman,	XXIV	XXV	XXVI	XXVII	XXVIII
Common,	24	25	26	27	28

Roman,	XXIX	XXX	XXXI	XXXII	XL
Common,	29	30	31	32	40

Roman,	L	LX	LXX	LXXX	XC	C	D	M
Common,	50	60	70	80	90	100	500	1000

Read the following numbers:

1. XIX. 2. XLV. 3. XCII. 4. LXV.
5. XXVII. 6. MCCC. 7. LXXIV. 8. MDCC.
9. XXIX. 10. MDV.

Write in Roman numerals:

11. 25. 12. 54. 13. 60. 14. 75. 15. 150.

If you study the Roman notation a few minutes, you will see that it is founded upon simple rules.

1. Repeating a letter repeats its value. III.

2. Placing a letter before one of greater value, its value is taken from the greater value. IV.

3. Placing a letter after one of greater value, its value is to be added to the greater value. VI.

4. Placing a letter between two letters, each of greater value, its value is to be taken from their sum. XIX.

SUMMARY

26. 1. We have learned to read numbers. We point off large numbers into groups or periods of three figures each, beginning at the last figure, and pointing off toward the left.

We read the figures of each group separately and give them the group name before we read the numbers of the next group.

The last group need not be named, but we may call it units.

The next higher groups at the left are: **thousands, millions, billions**.

In writing numbers, we write the numbers of each group as we have learned to write the numbers of the lowest group in the lower grades.

2. We have learned to read and to write dollars and cents.

We use a sign for dollars, and separate the dollars and cents by a period. We always use two places for the cents, writing in a cipher if there is only one figure in the number of cents given.

3. We have learned to add columns of large numbers. We keep the last line of figures straight up and down, even if the numbers are of different lengths, and are careful to write each figure directly under its corresponding figure in the number above it. We begin to add with the last column. If the sum of the figures in any column has more than one figure in it, we write down the last figure of the sum, and carry the other to be added with the next column.

4. We have learned to subtract a smaller number from a larger one, which we do by finding the number which we must add to the smaller one to make the larger number.

We have learned to subtract as business men do. It is like addition.

5. We have learned to add and to subtract dollars and cents. It is like adding or subtracting other numbers, only we have to be careful to keep the periods which separate the dollars and cents directly under each other.

6. We have learned that numbers must be of the same kind before we can add or subtract them.

7. We have learned to find the amount of change which a clerk should give us when we buy things and give him more than enough money to pay for them. We have learned to do it quickly, as the business men do.

8. We have learned to work examples, and can tell when to add and when to subtract to answer the questions in the examples.

To be remembered.

1. Reading numbers is called **numeration.**
2. Writing numbers is called **notation.**
3. The sign for dollars is $.
4. Only **like** numbers can be added or subtracted.
5. The sign for addition is +. It is read **plus.**

6. The sign for subtraction is −. It is read **minus.**

7. The answer in addition may be called the **sum.**

8. The answer in subtraction may be called **remainder.**

9. In writing cents, we sometimes use the sign ¢ and sometimes write cents as **parts of** dollars, using the dollar sign and the period. Five cents may be written 5¢ or $.05.

MULTIPLICATION

27. We have learned something about multiplication in the lower grades and have learned some of the tables. If we saw those tables written in the old-fashioned way, I am afraid we will not know them when we try to use them in examples where they are written in a very different way.

$2 \times 1 = 2$
$2 \times 2 = 4$
$2 \times 3 = 6$

See how many of them you know when they are written in tables as we have learned to write them in our new books, and as they will be used in examples.

$$\begin{array}{r} 2 \\ \times\, 2 \\ \hline 4 \end{array} \qquad \begin{array}{r} 3 \\ \times\, 2 \\ \hline 6 \end{array}$$

You read these either up or down, and should be able to read them either way. Read 2 2's are 4; 2 3's are 6; and 3 2's are 6.

It is important to read them both ways, as it makes the tables very short and easy. If you learn them both ways, this is all the multiplication table you will have to learn:

Table of 2's.

$$\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \times 2 & \times 2 & \times 2 & \times 2 & \times 2 & \times 2 & \times 2 & \times 2 & \times 2 \\ \hline 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \end{array}$$

Table of 3's.

$$\begin{array}{rrrrrrrr} 1 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \times 3 & \times 3 & \times 3 & \times 3 & \times 3 & \times 3 & \times 3 & \times 3 \\ \hline 3 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \end{array}$$

Table of 4's.

$$\begin{array}{rrrrrrr} 1 & 4 & 5 & 6 & 7 & 8 & 9 \\ \times 4 & \times 4 & \times 4 & \times 4 & \times 4 & \times 4 & \times 4 \\ \hline 4 & 16 & 20 & 24 & 28 & 32 & 36 \end{array}$$

Table of 5's.

$$\begin{array}{rrrrrr} 1 & 5 & 6 & 7 & 8 & 9 \\ \times 5 & \times 5 & \times 5 & \times 5 & \times 5 & \times 5 \\ \hline 5 & 25 & 30 & 35 & 40 & 45 \end{array}$$

Table of 6's.

$$\begin{array}{rrrrr} 1 & 6 & 7 & 8 & 9 \\ \times 6 & \times 6 & \times 6 & \times 6 & \times 6 \\ \hline 6 & 36 & 42 & 48 & 54 \end{array}$$

Table of 7's.

$$\begin{array}{rrrr} 1 & 7 & 8 & 9 \\ \times 7 & \times 7 & \times 7 & \times 7 \\ \hline 7 & 49 & 56 & 63 \end{array}$$

Table of 8’s.

$$\begin{array}{rrr} 1 & 8 & 9 \\ \times 8 & \times 8 & \times 8 \\ \hline 8 & 64 & 72 \end{array}$$

Table of 9’s.

$$\begin{array}{rr} 1 & 9 \\ \times 9 & \times 9 \\ \hline 9 & 81 \end{array}$$

The following are the hard ones to remember. Look at them often.

$$\begin{array}{rrrrrrr} 7 & 8 & 9 & 7 & 8 & 7 & 8 \\ \times 3 & \times 3 & \times 3 & \times 4 & \times 4 & \times 6 & \times 6 \\ \hline 21 & 24 & 27 & 28 & 32 & 42 & 48 \end{array}$$

$$\begin{array}{rrrrrrr} 9 & 7 & 8 & 9 & 8 & 9 & 9 \\ \times 6 & \times 7 & \times 7 & \times 7 & \times 8 & \times 8 & \times 9 \\ \hline 54 & 49 & 56 & 63 & 64 & 72 & 81 \end{array}$$

28. Read the following exercises as rapidly as possible. As they are all in multiplication, you do not need the sign to tell you what to do. After you have read them from the top number down, see if you can read each one, beginning at the lower number and reading up.

1\.
$$\begin{array}{cccccccccccccc} 7 & 9 & 7 & 8 & 9 & 6 & 4 & 3 & 2 & 5 & 7 & 9 & 5 & 4 \\ 3 & 4 & 5 & 4 & 2 & 3 & 5 & 3 & 2 & 6 & 3 & 2 & 3 & 4 \\ \hline \end{array}$$

Read them, 7 3’s are 21; 9 4’s are 36; 7 5’s are 35, etc. Reading up, we read, 3 7’s are 21; 4 9’s are 36, etc.

2.	9	7	8	7	6	4	2	8	9	7	4	3
	2	2	3	2	2	7	8	3	2	4	6	9

3.	8	8	7	7	7	10	9	4	3	8	7
	3	4	3	6	7	5	6	7	9	3	4

4.	6	9	7	5	6	9	7	4	6	7	6
	2	2	2	3	4	2	3	6	4	3	5

5.	5	8	6	9	7	8	4	9	9	6	7	8	9	7	4
	8	5	9	6	8	7	7	4	6	9	8	7	7	9	4

6.	7	9	5	4	7	5	9	7	5	3	9	7	8
	9	7	4	5	5	7	7	9	3	5	7	9	3

7.	7	8	9	7	8	7	8	9	7	8	9	4	6
	8	9	7	8	7	8	9	7	8	9	4	6	7

29. 1. If one chair costs $7, what will 8 chairs cost?

2. If a man earns $3 per day, how much will he earn in 8 days?

3. There are 4 quarts in one gallon. How many quarts in 7 gallons?

4. There are 3 feet in one yard. How many feet in 9 yards?

5. If a boy earns 7 ¢ each hour, how much will he earn in 8 hours?

6. There are 7 days in a week. How many days in 9 weeks?

7. A man paid $3 apiece for barrels of apples. He bought 9 barrels. What did they cost him?

If he gave the man $30 to pay for the apples, how much change should he receive?

8. Find the cost of the following bill of goods:

3 oranges at 4 ¢
8 apples at 2 ¢
6 peaches at 3 ¢

After you find the whole cost, see how much change you would receive if you gave the man one dollar.

9. Find the total cost of 3 bushels of chestnuts at $2 per bushel; 6 barrels of apples at $3 per barrel; and 4 quarts of walnuts at 9 ¢ per quart.

10. A man earns $15 per week, and spends $9 per week for board. How much does he save each week? How much will he save in 4 weeks?

MULTIPLICATION

30. With multiplier of one figure.

1. Cost of 7 acres of land at $ 125 per acre?

$$\begin{array}{r}\$125\\ \times 7\\ \hline \$875\end{array}\quad \textit{Ans.}$$

We read the work, 7 5's are 35, but write only the 5 in the column under the multiplier, and carry the 3 as we have learned to do in our work in addition, and keep it in mind to be added to the next column. Then we read the next column, 7

2's are 14, and adding the 3 which we have kept in mind for this column, we have 17. We write the 7 and carry the 1, and read the next column, 7 1's are 7, and add the 1 which we have carried, writing 8. The answer is found to be 875, and as the answer is always the same kind as the number which we multiply, we know that it is dollars, and write $875.

2. Find the cost of 9 horses at $175 each.

$$\begin{array}{r} \$175 \\ \times 9 \\ \hline \$1575 \end{array}\ \textit{Ans.}$$

Read it, 9 5's 45; 9 7's 63 and 4, 67; 9 1's 9 and 6, 15.

3. How many bushels in 7 bins, each bin holding 253 bushels?

$$\begin{array}{r} 253 \\ \times 7 \\ \hline 1771 \end{array}$$

253 bushels
× 7
1771 bushels. *Ans.*

Read it, 7 3's 21; 7 5's 35 and 2, 37; 7 2's 14 and 3, 17. Why is the answer bushels?

4. Find the cost of 8 barrels of apples at $3.50 per barrel.

$$\begin{array}{r} \$3.50 \\ \times 8 \\ \hline \$28.00 \end{array}\ \textit{Ans.}$$

Read it, 8 0's 0; 8 5's 40; 8 3's 24, 28.

5. A man earns $2.50 per day. How much will he earn in one week, working 6 days?

6. Find the total cost of 8 cows at $45 each and 4 horses at $185 each.

$45	$185	$360
×8	×4	+740
$360 Cost of cows.	$740 Cost of horses.	$1100 *Ans.*

7. Find the cost of 7 tons of coal at $6.50 per ton, 8 loads of wood at $2.75 per load, and some kindling wood for $1. (Find the cost of coal and wood, and then add cost of coal, wood, and kindling wood to get total cost.)

8. Bought 6 lb. sugar at 6¢ per pound, 7 lb. rice at 8¢, 2 lb. coffee at 36¢. What did they all cost? If I pay two dollars, how much change should I receive?

9. Find the cost of the following bill:

3 pounds at 12¢
8 pounds at 15¢
7 pounds at 25¢
9 pounds at 60¢

If I pay $10, how much change should I receive?

10. (The sign lb. means pounds.)

Find the cost of 9 lb. at 16¢; 7 lb. at 12¢; 8 lb. at 9¢; 4 lb. at 50¢; 2 lb. at 75¢. If I gave the man a ten-dollar bill, how much change should I receive?

Note. The names of the articles bought are left out, that you may see that it makes no difference whether we are buying candy, marbles, or butter; if the price and the amount bought are given, we can find the cost.

31. When there are more figures than one in the multiplier.

1. Find the cost of 32 sheep at $3.50 each.

$$\begin{array}{r} \$3.50 \\ \times\ 32 \\ \hline 7\ 00 \\ 105\ 0 \\ \hline \$112.00 \end{array} \quad \textit{Ans.}$$

It is just as easy to multiply by two figures as by one, if we multiply by each figure separately, and begin to write the result directly under the figure of the multiplier which we are using. When we begin to multiply by the 3, we begin to write the result directly under the 3. Then we add the two results, and get the full answer to our question.

2. Multiply 6732 by 76.

$$\begin{array}{r} 6732 \\ \times\ 76 \\ \hline 40392 \\ 47124 \\ \hline 511632 \end{array} \quad \textit{Ans.}$$

3. Find the cost of 12 acres at $85 per acre.

4. Find the cost of 65 bushels of oats at 45¢ per bushel.

5. Find the number of bushels of potatoes on 15 acres, if there are 95 bushels on each acre.

6. Find the cost of 24 sheep at $3.75 each.

7. Multiply 6748 by 76.

8. Find the total cost of 12 horses at $175 and 30 cows at $65.

9. Find the cost of 25 tons of hay at $18 per ton and 15 bushels of oats at $.45 per bushel.

10. Bought shoes for $3.50; rubbers for $.90; and 2 pairs of slippers at $1.25 a pair. What did they all cost?

32. When the multiplier has a cipher at the end.

1. Find the cost of ten barrels of apples at $3.75 per barrel.

$$\begin{array}{r} \$3.75 \\ \times 10 \\ \hline \$37.50 \end{array}$$

As we do not have to multiply by the cipher, because it has no value, we can place it outside of the line of figures in the number to be multiplied, and bring it down so as to get the right number of figures in the answer.

Notice how much this saves. If we write it under the other figures, and use it as a figure to multiply by, we have this:

$$\begin{array}{r} \$3.75 \\ \times 10 \\ \hline 0\ 00 \\ 37\ 5 \\ \hline \$37.50 \end{array}$$

The answer is the same, but we need not write out all of the ciphers.

2. Find the cost of 20 tons of hay at $18 per ton.

3. Find the cost of 60 sheep at $4.50 each.

4. What will 70 bushels of potatoes cost at $1.25 per bushel?

5. Multiply 784 by 20; 675 by 30; 743 by 40; 452 by 50.

6. Multiply 875 by 60; 764 by 70; 724 by 80; 765 by 90.

33. Multiplying by 10, 100, 1000, etc.

1. Multiply 67 by 10.

Long way	Short way
67 × 10 670	67 with 0 annexed. 670.

2. Multiply 854 by 10. Multiply 765 by 10.

3. Multiply 54 by 100. (Annex two ciphers.) 5400 *Ans.*

4. Multiply 784 by 100. Multiply 678 by 100.

5. Multiply 675 by 1000. (Annex three ciphers.) 675000 *Ans.*

6. Multiply 854 by 1000. 682 by 1000. 45 by 1000.

Write the answers at sight.

(7)	(8)	(9)	(10)
65 × 100.	75 × 1000.	328 × 10.	67 × 100.

When we use the short way to multiply dollars and cents, we must be careful to point off only two places for cents in the answer.

11. Multiply $65.15 by 10. $651.50 *Ans.*

12. Multiply $25.85 by 100. $2585. *Ans.*

13. $12.25 × 10. (Read $12.25 multiplied by 10.)

14. 10 × $12.15. (Read 10 times $12.15.) (10 is the multiplier.)

34.

$$\begin{array}{rl} 457 & \text{(multiplicand)} \\ 23 & \text{(multiplier)} \\ \hline 1371 & \\ 914 & \\ \hline 10511 & \text{(product)} \end{array}$$

See how the names are used.

1. The multiplicand is 65, the multiplier is 15. What is the product?

2. If the multiplicand is 845, and the multiplier 12, what is the product?

3. Find the cost of 60 sheep at $3.50 each. Which number is the multiplier? Which is the multiplicand? Will the answer be the number of sheep or the number of dollars? If you cannot tell, see how the next example is worked out.

4. Find the cost of 45 cows at $65 each.

$$\begin{array}{rl} \$65 & \\ 45 & \\ \hline 325 & \\ 260 & \\ \hline \$2925 & \textit{Ans.} \end{array}$$

The question asks how many dollars, and we make the multiplicand the same kind as the answer sought. The multiplier must always be considered an abstract number. We think, if 1 cow costs $65, 45 cows will cost 45 times $65, or $2925.

5. Find the cost of 75 bushels of oats at $.45 per bushel.

Analysis: 1 bushel costs $.45; 75 bushels cost 75 times $.45.

6. If one horse costs $175, what will 8 horses cost?

Analysis: 1 horse costs $175; 8 horses cost 8 times $175.

MULTIPLICATION

35. Gathering up what we have learned in multiplication.

1. The sign for multiplication is ×.

2. The number which we multiply is called the **multiplicand.**

3. The number which we use to multiply by is called the **multiplier.**

4. The result obtained by multiplication is called the **product.** It is always the same kind as the multiplicand.

5. The multiplicand may be the number of any kind of units, such as dollars and cents, marbles, oranges, cows, etc.

6. The multiplier cannot name any units, as it tells how many times we use the units of the multiplicand. It is called **abstract,** as it does not name any kind of units.

7. We write the multiplier under the multiplicand, and write the first figure which we find of the

product directly under the figure we are using of the multiplier.

8. Where there are more figures than one in the multiplier, we use each figure separately, and write the product for each figure used, beginning it directly under the figure used in the multiplier. We then add the products.

9. When there is a cipher at the end of the multiplier, we may write the multiplier so that the cipher stands at the right of the figures in the multiplicand, and write a cipher in the product directly under it, then multiply as usual. This is not necessary, but it is easier.

10. To multiply by 10, we annex one cipher to the multiplicand; to multiply by 100, we annex two ciphers; and to multiply by 1000, we annex three ciphers, and have the product.

11. We multiply dollars and cents just like other numbers, and point off two figures for cents in the product.

12. We cannot multiply by dollars and cents, for the multiplier must always be an abstract number.

13. The sign × for multiplication is read **times** when the multiplier precedes it, and **multiplied by** when the multiplier follows it. 3 × $ 2 is read three times $ 2. $ 2 × 3 is read $ 2 multiplied by 3.

36. General Review — Addition, Subtraction, and Multiplication.

1. Two boys run a race. At the call to stop, one boy has run 450 yards and the other 396 yards. How far apart are they?

2. In a school of 2376 pupils, 1876 of the pupils are girls. How many boys are there?

3. A man gathered 2500 pounds of rice. He sold 1750 lb. What was the value of the part left, at 9 ¢ a pound?

4. A man had 1172 books in his library, the books being in three cases. In one case there are 375 books, in another 450 books. How many books in the third case?

5. A dealer had 204 bushels of potatoes. He sold 76 bushels to one man and 108 bushels to another. What was the value of the remainder, at 80¢ a bushel?

6. Which would you rather have, \$ 240 + \$ 356, or \$ 983 – \$ 195?

7. I have \$ 236. How much more must I earn to have \$ 500?

8. A girl bought a book for \$ 1.25, a pen for 49¢, and a bottle of ink for 15¢. How much change should she receive, if she paid the clerk \$ 2?

9. A man paid \$ 4500 for a house, and then

paid $175 to have it painted. If he should then sell it for $5000, how much would he gain?

10. A boy who had $25.63 wished to buy a bicycle which cost $65. How much more money does he need?

11. At a fair they used 95 lb. of candy the first night and 120 lb. the second night. How many pounds did they use?

12. Find what I paid for 17 tons of coal at $6.50 and 9 cords of wood at $4.50 per cord.

13. At 27¢ a pound, what will a 14-lb. turkey cost?

14. A freight car will carry 14,115 lb. of coal. What will 36 such cars carry?

15. Multiply the sum of 218 and 946 by 7.

16. Add the following: $975; $874; $24.50; $75.60; $52.40; $35.25; $150.24; $45.18; $54.72.

17. A man's income is $160 a month; his expenses are $95 a month. How much will he save in one year?

18. If it require 125 tons of iron rails for each mile of railroad, how many tons will it require for 76 miles?

19. Add the following: 34,765; 4576; 89,567; 34,526; 4326; 34,765; 89,754; 45,321.

(20)	(21)	(22)
3456702	$3542.17	234576
4678	3645.70	2385463
345601	23.76	16789
321	16743.20	3456
67890	75.00	4532
30123	23456.00	1876
54321025	4578.56	9876

DIVISION

37. We have learned that 3 8's are 24; now we are to look at the same thing from a different point of view, and ask the question, 24 is how many 8's?

In answering this question in form of division, we write it,

$\begin{array}{r} 3 \\ 8\overline{)24} \end{array}$ Read it, 24 is 3 8's.

We begin at the 24, read up and around to the 8. In the question form, we read, 24 is how many 8's? In the answer to the question, we read, 24 is 3 8's.

$\begin{array}{r} 3 \\ 6\overline{)18} \end{array}$ Read, 18 is 3 6's.

$\begin{array}{r} ? \\ 7\overline{)21} \end{array}$ Read, 21 is how many 7's?

$\begin{array}{r} ? \\ 9\overline{)81} \end{array}$ Question form, 81 is how many 9's?

$\begin{array}{r} 9 \\ 9\overline{)81} \end{array}$ Answer form, 81 is 9 9's.

To the Pupils. Some books place the answer in division under the line like this, $\begin{array}{r}6\overline{)42}\\7\end{array}$ and read it, 6 in 42 7 times.

The exercises in this book can be arranged in that way if you wish; but it will be better for you to change and take the way given first, even if you have begun to read them in the old-fashioned way.

If you learn to write the answer in the proper place in this work, we shall not have to change when we come to use large numbers, or when we come to use decimals. If you learn to read it the right way, beginning at the large number, called the **dividend**, and read **up** through the answer to the **divisor**, you will find the work much easier, and when you come to use numbers which are not divided without remainders, you will have no trouble.

You will find that you can read large numbers like this

$$\begin{array}{r}7243\\6\overline{)43458}\end{array}$$

easily if you learn to begin at the right place. This is read, 43 is 7 6's and 1; 14 is 2 6's and 2; 25 is 4 6's and 1; 18 is 3 6's.

We can then check our work, and see if we have made no mistake by using the 6 as a multiplier, and multiplying the answer to see if it gives the number under the line. We do not have to write them again, but can check by reading, 6 3's 18; 6 4's 24, 25; 6 2's 12, 14; 6 7's 42, 43.

38. In dividing dollars and cents, we use the same form:

$$\begin{array}{r}\$\ 3.10\\6\overline{)\$18.60}\end{array}$$

Read it, 18 is 3 6's; 6 is 1 6; 0 is no 6; and place the mark separating dollars and cents **directly over** the one in the **dividend.**

See if you can read the following in the best way:

1. $$\begin{array}{rrrr}61 & 31 & 41 & 21221\\7\overline{)427} & 8\overline{)248} & 9\overline{)369} & 4\overline{)84884}\end{array}$$

2. $\begin{array}{r} 423242234 \\ 2\overline{)846482468} \end{array}$ Read, 8 is 4 2's; 4 is 2 2's; 6 is 3 2's; 4 is 2 2's; 8 is 4 2's; 2 is 1 2; 4 is 2 2's; 6 is 3 2's; 8 is 4 2's.

3. $\begin{array}{r} 21002 \\ 7\overline{)147014} \end{array}$ Read it, 14 is 2 7's; 7 is 1 7; 0 is no 7; 1 is no 7; 14 is 2 7's.

39. The sign for division is ÷. 12 ÷ 6 means that we are to find how many 6's in 12.

40. Write the following in the proper form, and read them in the way we have just explained:

1. 24 ÷ 6. Write it, $\begin{array}{r} 4 \\ 6\overline{)24} \end{array}$. Read it, 24 is 4 6's.
2. 81 ÷ 9. 64 ÷ 8. 72 ÷ 8. 63 ÷ 7. 24 ÷ 4.
3. 36 ÷ 9. 54 ÷ 9. 49 ÷ 7. 63 ÷ 7. 27 ÷ 3.
4. 56 ÷ 8. 48 ÷ 6. 48 ÷ 8. 28 ÷ 4. 28 ÷ 7.
5. 21 ÷ 3. 36 ÷ 4. 21 ÷ 7. 36 ÷ 9. 35 ÷ 7. 16 ÷ 2.

Arrange the following in proper written form, and read them in the best way.

41. 1. If a boy pays 6¢ for 2 marbles, how much does he pay for each marble?

Write it $\begin{array}{r} 3\text{¢} \\ 2\overline{)6\text{¢}} \end{array}$ Read it, 6 is 3 2's. Then give the answer the **same name** as the **dividend**, as the answer is the **same** kind of **units as** the **dividend**.

2. Paid $24.68 for 2 plows. How much did I pay for each?

Written

$$\begin{array}{r} \$12.34 \\ 2\overline{)\$24.68} \end{array}$$

Read, 2 is 1 2; 4 is 2 2's; 6 is 3 2's; 8 is 4 2's. Then we write the **period directly** over the one in the **dividend,** and use the sign for dollars, thus giving the answer the same name for its units as the dividend.

3. If I pay $84.48 for 4 tons of hay, how much do I pay for each ton?

4. If 5 tons of coal cost $30.50, what does each ton cost?

5. If there are 690 acres in three farms, each of the same size, how many acres are there in each farm?

42. Sometimes the divisor is of the same kind as the dividend, and then the answer is **abstract.** We have learned in multiplication, which is the reverse of division, that one of the numbers, the multiplier, must be abstract. In division, either the answer or the divisor **must** be abstract. When the divisor is abstract, the answer is like the dividend. When the divisor is not abstract, the answer **must be abstract.**

1. If oranges are 5¢ each, how many can I buy for 20¢?

Written

$$\begin{array}{r} 4 \\ 5¢\overline{)20¢} \end{array}$$

Explained: 20 is 4 5's. *Ans.* 4. The answer is not 4 oranges, for we cannot say 20¢ is 4 oranges times 5¢, but can say 20¢ is 4 times 5¢.

2. When apples are 2¢ apiece, how many can I buy for 20¢?

Written

$$\begin{array}{r}10\\2\text{¢}\overline{)20\text{¢}}\end{array}$$

Ans. 10. Read, 2 is 1, 2; 0 is no 2's.

3. Find the cost of each sheep when we pay $50 for 5 sheep.

4. Find the cost of pencils when we pay 8¢ for 4 pencils.

43. When the division of each figure is not exact, we use the remainder in its proper place before the next figure of the dividend, and continue our division.

$$\begin{array}{r}18\\3\overline{)36}\end{array}$$

Read, 3 is 1 2 and 1; then use the 1 in its regular place before the 6, and read 16 is 8 2's.

1. $\begin{array}{r}13\\4\overline{)52}\end{array}$ Read, 5 is 1 4 and 1; 12 is 3 4's.

2. $\begin{array}{r}13\\6\overline{)78}\end{array}$ Read, 7 is 1 6 and 1; 18 is 3 6's.

3. $\begin{array}{r}27\\3\overline{)81}\end{array}$ Read, 8 is 2 3's and 2; 21 is 7 3's.

4. $\begin{array}{r}2747\\3\overline{)8241}\end{array}$ Read, 8 is 2 3's and 2; 22 is 7 3's and 1; 14 is 4 3's and 2; 21 is 7 3's.

5. $\begin{array}{r}937\\9\overline{)8433}\end{array}$ Read, 84 is 9 9's and 3; 33 is 3 9's and 6; 63 is 7 9's.

Note to the Teacher. Drills with numbers written on the blackboard, where pupils can read them rapidly, should be given to enable them to determine the quotient figure and the remainder without hesitation. Early drills may be in column form, with single division, where answer to 13 divided by 3 is read, and give answer only. In writing these drills on the board, use the form

$$3\overline{)13}$$

because of its value in the visualization habits of the pupils. Tell them to read the answers only, and the class will read 4 and 1 as the short form. Then take long numbers, using small divisors for drill on the tables, such as

$$3\overline{)4132607843267 56}$$

and guide the reading by the pointer, writing in the answer as the class reads, figure by figure, 4 is 1 3 and 1; 11 is 3 3's and 2; 23 is 7 3's and 2; 22 is 7 3's and 1; 16 is 5 3's and 1; 10 is 3 3's and 1, etc.

As they use the remainder at once, there is no danger of forgetting it, nor need of writing it at the side, as we used to do with the old form in "short" division.

44. Work the following, reading them aloud as you work them:

1. $2\overline{)3624}$
2. $3\overline{)4518}$
3. $4\overline{)14624}$
4. $5\overline{)37015}$
5. $6\overline{)7428}$
6. $7\overline{)352163}$
7. $8\overline{)24712}$
8. $9\overline{)36918}$

Write out many others on your papers, and learn to read them quickly, and to write the answers as you read.

45. 1. If 1 book costs $ 2, how many can you buy for $ 7? How much money will you have left?

2. Oranges are 4¢ each. How many can you buy for 15¢? How much money left?

3. Pencils are 3¢ each. How many can you buy for 20¢? How much money left?

4. A man has $50. How many sheep at $6 each can he buy? How much money left?

5. If a railroad ticket to New York costs $6, how many such tickets can be bought for $50? How much money left?

6. How many four-quart measures can you fill from 30 quarts of milk? How many quarts of milk will you have left?

7. At $9 each, how many calves can a farmer buy for $75? How much money left?

8. Coal is $6 per ton. How many tons can I buy for $40? How much money left?

9. I have 60 chestnuts to divide among 9 boys. How many can I give to each, and have each receive the same number? Shall I have any left?

10. Flour is $8 per barrel. How many barrels can I buy for $30? How much money left?

46. Introduction to analysis and to use of type forms.

1. If 3 oranges cost 6¢, what will 5 oranges cost?

3 oranges cost 6¢,
1 orange costs 2¢,
5 oranges cost 10¢. *Ans.*

2. If 9 books cost $2.25, what will 7 books cost?

9 cost $2.25,
1 costs $ .25,
7 cost $1.75. *Ans.*

3. If 6 pounds of tea cost $3.60, what will 9 pounds of tea cost?

4. If 9 calves cost $72, what will 25 calves cost?

5. If I can buy 4 tons of coal for $26, what will 6 tons of coal cost?

6. Paid $48 for 8 chairs. At the same price, what will 12 chairs cost?

7. Paid $4.50 for 6 books. At the same price, what will 8 books cost?

8. If 8 cows cost $384, what will 25 cows cost at the same rate?

9. If I earn $20 in 8 days, how much can I earn at the same rate in 20 days?

10. I buy 6 calves for $51. What will 28 calves cost at the same rate?

47. Exercises in writing examples.

Write out examples of your own from the following facts:

1. 3 —— cost 9¢,
5 —— cost ?¢

You can make many examples from this form, such as: If 3 pencils cost 9¢, what will 5 pencils cost? or: Paid 9¢ for 3 pads. What did I pay for 5 pads? or: If I can buy 3 papers for 9¢, what should I pay for 5 papers?

2. 4 —— cost $ 8,
7 —— cost $?

3. 5 —— cost 75¢,
7 cost ?¢

4. 9 —— cost 45¢,
24 cost $?

5. 6 cost $ 54,
17 cost $?

6. 4 cost $ 36,
20 cost $?

7. 6 cost $ 48,
15 cost $?

8. 7 cost 42¢,
38 cost ?¢

9. 7 cost 56¢,
15 cost ?¢

10. 8 cost $ 56,
45 cost $?

48. Division with two or more figures in the divisor.

It is quite hard to divide when we have large numbers in the divisor. People used to call this "Long Division."

It is done just as we have been dividing, only when the numbers are large we cannot do all of the work mentally, and we have to write some of it down. We think of it in parts, using only a part of the dividend at a time, just as we have been doing when we had large numbers in the dividend.

Compare very carefully the two ways given below:

Short form

$$\begin{array}{r} 246 \text{ } \mathit{Ans.} \\ 3\overline{)738} \end{array}$$

Long division form

$$\begin{array}{rl} & 246 \text{ } \mathit{Ans.} \\ 3) & \overline{738} \\ & \underline{6} \\ & 13 \\ & \underline{12} \\ & 18 \\ & \underline{18} \end{array}$$

In the short way, we carry in our mind, and think of the next number.

In the long way, we write down each step, and do not try to remember what to carry or to think of the next number.

You read the short form, just as we have written out the long form: 7 is 2 3's and 1; 13 is 4 3's and 1; 18 is 6 3's.

If you study the long form, you will see that we take **four steps**, as follows:

1. See how many, and write it above as a part of the quotient.

2. Multiply the divisor by this figure.

3. Subtract the product from the part of the dividend which we are using.

4. Annex the next part of the dividend to the remainder.

49. 1. Divide 945 by 21.

$$\begin{array}{rl} & 45 \text{ } \mathit{Ans.} \\ 21) & \overline{945} \\ & \underline{84} \\ & 105 \\ & \underline{\underline{105}} \end{array}$$

Tell what you did, and see if you can find the **four** steps.

2. Divide 9641 by 31.
3. Divide 8856 by 41.
4. Divide 6573 by 21.
5. Divide 10,914 by 51.
6. Divide 6908 by 22.
7. Divide 15,552 by 72.
8. Divide 36,244 by 81.
9. Divide 100,533 by 31.
10. Divide 159,273 by 51.

Note. **At first, you may cover all but the left-hand figure of your divisor, and think of that when you take the first step.**

In example (1) given above, cover all but the 2 of the divisor, and think 9 is how many 2's, instead of 94 is how many 21's. The first step is the hardest one of all, and you may need help about that one. The other steps are easy.

50. One or more figures in the divisor.

1. If 12 horses cost $1560, what will one horse cost?

2. How many oranges, at 4¢ each, can I buy for 24 cents?

3. How many acres, at $75 an acre, can I buy for $22,575?

4. There are 4 quarts in one gallon. How many gallons in 8 quarts?

5. There are 24 hours in one day. How many days in 3600 hours?

6. In 64 loads of hay there were 143,360 pounds. How many pounds in each load, if they were each of the same weight?

7. Paid $702 for 117 tons of coal. How much did I pay for each ton?

8. Paid $702 for coal at $6 per ton. How many tons did I buy?

9. If 12 barrels of flour cost $132, what will 48 barrels cost?

10. If 13 horses cost $2275, what will 25 horses cost at the same rate?

11. What will 8 cows cost, at the rate of 50 cows for $1250?

12. Find the cost of 48 pounds of butter if 81 pounds cost $25.92.

13. Exchanged 50 bushels of potatoes, at $.75 per bushel, with the grocer for 5 barrels of flour. What was the price of flour per barrel?

51. When the dividend cannot be fully divided.

1. Divide 12 by 5.

$$\begin{array}{r} 2\frac{2}{5} \quad \textit{Ans.} \\ 5\overline{)12} \phantom{\quad \textit{Ans.}} \end{array}$$

When we have a remainder, after dividing as far as we can, we write the remainder over the divisor, which shows that that part of the dividend has not been divided. We shall learn more about these unfinished divisions a little later.

2. Divide 3435 by 4. $\begin{array}{r} 858\frac{3}{4} \\ 4\overline{)3435} \end{array}$

3. Divide 32,146 by 31.
4. Divide 62,141 by 61.
5. Divide 3462 by 32.
6. Divide 876 by 9.
7. Divide 34,045 by 14.
8. Divide 4327 by 45.
9. Divide 3862 by 34.
10. Divide 7642 by 46.
11. Divide 3460 by 10.
12. Divide 4370 by 10.
13. Divide 52,600 by 100.

52. Short process, when there are ciphers at the end of the divisor.

1. $4350 \div 10.$ $\begin{array}{r} 435 \\ 10\overline{)4350} \\ \underline{40} \\ 35 \\ \underline{30} \\ 50 \\ \underline{50} \end{array}$ Short way, $\begin{array}{r} 435 \\ 1\cancel{0}\overline{)435\cancel{0}} \end{array}$

We need not use the cipher in the divisor. If we will cross off the same number of figures in the dividend as we cross off ciphers at the end of the divisor, we can use the remaining parts of the divisor and of the dividend and get the same result as if we had used the full divisor and the full dividend.

53. Write the answers at sight.

$960 \div 10$ $840 \div 10$ $720 \div 10$ $640 \div 20$
$360 \div 20$ $420 \div 20$ $960 \div 30$ $840 \div 40$
$750 \div 50$ $920 \div 10$ $380 \div 20$ $720 \div 30$
$800 \div 200$ $1200 \div 600$ $450 \div 90$ $1800 \div 900$

54. If we divide 843 by 10, we have 84 and 3 as a remainder; and write the answer $84\frac{3}{10}$. Working it the short way, $84\cancel{3} \div 1\cancel{0}$, we see that the number crossed off is the remainder.

Write answers at sight.

424 ÷ 10 *Ans.* $42\frac{4}{10}$ 623 ÷ 20 *Ans.* $31\frac{3}{20}$
421 ÷ 10   517 ÷ 10   846 ÷ 10
763 ÷ 100 *Ans.* $7\frac{63}{100}$ 849 ÷ 100
4324 ÷ 100   874 ÷ 10   9634 ÷ 100
5426 ÷ 100   764 ÷ 20   841 ÷ 20
493 ÷ 70   640 ÷ 20   643 ÷ 80   281 ÷ 70
420 ÷ 60   424 ÷ 70   841 ÷ 200   275 ÷ 30
3683 ÷ 900   4817 ÷ 800
5432 ÷ 600   5000 ÷ 1000
35 × 100   43 × 20   36 × 100   48 × 1000
212 × 1000   24 × 200   28 × 200   12 × 3000

SUMMARY OF DIVISION

55. 1. Division is reversed multiplication.

2. The number to be divided is called the dividend. It is like the product in multiplication.

3. The number by which we divide is called the divisor. It is like a factor in multiplication.

4. The result obtained by division is called the quotient. It is like a factor in multiplication.

5. Both the divisor and the quotient may be abstract. One of them must be abstract.

6. The number remaining when the division is not exact is called the remainder.

7. If we multiply the divisor and the quotient together and add the remainder, the result will equal the dividend. We can test our work in division by doing this.

8. In dividing numbers of dollars and cents, if the divisor is abstract, we point off the quotient by placing the point separating the dollars and cents directly over the point in the dividend. If the divisor is dollars and cents, the answer will be abstract, and we divide as in simple numbers.

9. When we have a large number for a divisor, we can write out the work, using four steps.

10. When there are ciphers at the end of the divisor, we can use a short way, if we are careful about writing the remainder.

11. We write the remainder above the divisor at the end of the quotient.

TYPE FORM OF PROBLEMS

56. So many examples take this form, " If —— cost $ ——, what will —— cost?" that it is sometimes called a " type form."

The written form for working examples of this type, which is generally best, is shown in the work for the next example.

1. If 38 lb. of sugar cost $1.90, what will 49 lb. cost?

38 lb. cost $1.90,
1 lb. cost $?
49 lb. cost $? *Ans.*

Another common form for examples is: "If —— cost $ ——, how many can be bought for $ ——?"

2. If 4 oranges cost 12¢, how many oranges can be bought for 18¢?

4 oranges cost 12¢,
1 orange costs ?¢,
? oranges cost 18¢.

or

12¢ buys 4 oranges,
18¢ buys ? oranges?

3. If 16 pounds of butter cost 96¢, how many pounds can you buy for $7.68?

4. If 24 barrels cost $96, how many barrels can you buy for $480?

5. If 2 barrels cost $8, how many barrels can you buy for $12?

Can you see that the 4th and 5th are alike, except that one has larger numbers?

Use the form: If —— cost ——, how many for ——? and write ten examples for your classmates. Do not write any so hard that you cannot work them yourself.

57. Type where cost or quantities may be compared, and where it is hard to find the cost of 1 first.

1. A man sells oranges at the rate of 3 for 10¢; what will he charge for 12 oranges?

3 oranges cost 10¢,
12 oranges cost ?¢.

Think, four times as many oranges, four times as much cost.

2. A grocer sells crackers, 7 lb. for 50¢; how many pounds can you buy for $1.50 at this rate?

3. If 16 lb. of sugar cost 84¢, what will 32 lb. cost?

4. If I can buy 3 lb. of tea for $1, how many pounds can I buy for $4?

5. 8 lb. cost 25¢,
16 lb. cost ? ¢.

6. 12 cost 15¢,
36 cost ?¢.

7. 50¢ buys 4 lb.
$1 buys ? lb.

8. $.75 buys 6 bushels,
$1.50 buys ? bushels?

9. 18 lb. butter cost $5,
? lb. butter for $15.

10. 6 for $.25,
? for $1.

Does it make any difference in the answer to the 10th question whether it is 6 oranges or 6 apples which are bought for 25¢?

Note. In the 5th, 6th, 7th, and some others, the name of the thing bought is left out, that the plan may be fixed.

GENERAL REVIEW

58. 1. Find the sum of 3472, 879, 7864, 3589, 6759, 78,453, 7865.

2. Paid $ 875.75 for some cotton, and sold it for $ 900. How much did I gain?

3. If 25 lb. of butter cost $ 7.50, what will 60 lb. cost?

4. A farmer sold 120 bushels of wheat at 90¢ a bushel. How many calves at $ 6 each can he buy with the money received?

5. If 9 tons of coal cost $ 54, what will 26 tons cost?

6. If the divisor is 55 and the dividend 10,285, what is the quotient?

7. A man can earn $ 1125 in 9 months. How many months will it take him to earn $750?

8. Find the cost of 250 tierces of lard, each weighing 340 lb., at 7¢ a pound.

9. Bought gloves for $ 1.25, a hat for $ 3.50, a tie for $.75, and shoes for $ 3.75. If I give the clerk two $ 5 bills, how much change should I receive?

10. A man gave $ 25 for a wagon and then paid $ 9 for repairing it. If he should sell it for $ 40, what would he gain?

11. A man pays $ 312 a year for the rent of his home. What is the monthly rate?

12. At $ 45 a month for rent, what will it cost to rent a house for 2 years?

13. A man sold some cattle at an average price of $ 65 each. He received $ 7168. How many cattle did he sell?

14. If 35 barrels of molasses cost $ 525, what will 24 barrels cost at the same rate?

15. An eighteen-acre field produced 720 bu. of oats. How many bushels to the acre did it yield?

16. If 75 ft. of garden hose cost $ 11.25, what will 125 ft. cost at the same rate?

17. How many days must a man work at $ 2.50 per day to earn $ 150?

18. At an average speed of 35 miles per hour, how many hours will it take a train to run 420 miles?

19. The divisor is 84, the quotient 108, and the remainder 16. Find the dividend.

20. What will 156 bushels of potatoes cost at $ 1.65 per bushel?

21. A steamer burns 453 tons of coal in 3 days. How many days will 1547 tons last at the same rate?

22. Find the total cost of 78 lb. codfish at 6¢; 56 lb. halibut at 18¢; 2 barrels herring at $ 3.75.

23. A railroad car will hold 19,500 pounds. How many barrels of cement, each weighing 285 lb., will it carry?

24. At $ 23.75 each, how many plows can be bought for $ 500? How much money will be left?

25. The distance from New York to Albany is 140 miles, and the fare is $ 2.80. What is the rate per mile?

26. Find the loss on 12 cows bought for $ 700 and sold at $ 55 each.

27. A barrel of flour contains 196 pounds of flour. How many barrels can be filled from 6076 lb. of flour?

28. What will be the cost of 240 pounds of wheat at 90¢ per bushel of 60 lb.?

29. A girl gives a $ 5 bill in payment for four 75-cent dolls. How much change should she receive?

30. If 38 yards of cloth cost $ 180.50, what will 36 yards cost at the same rate?

31. A man borrowed $ 3850. After paying $ 640 and $ 250, how much does he still owe?

32. What should be paid for 25 tons of coal, if 200 tons cost $ 1750?

33. Tea is 80¢ a pound. How many pounds can you buy for $ 25?

34. If 35 acres of land cost $ 1750, how much will 80 acres cost at the same rate?

35. A farmer raised 6427 bushels of oats on 107 acres of land. What was the yield per acre?

36. How many boats, each carrying 8700 bushels of wheat, will it take to carry 843,900 bushels of wheat?

37. A farmer had 800 bushels of wheat. How much had he after selling 8 loads of 70 bushels each?

38. By selling a house for $ 5750, a man made a profit of $ 250. What did the house cost him?

39. Find the cost of 60 oranges at the rate of 3 oranges for 5 ¢.

40. A farmer exchanges 12 sheep, worth $ 15 each, for cows worth $ 45 each. How many cows should he receive?

41. How long will it take a man to do a piece of work which 12 men can do in 9 days?

42. 1400 cabbage plants were planted in 40 rows. How many plants were there in each row?

43. A man buys a piano for $ 750, paying $ 525 cash. How long will it take him to pay the remainder at $ 25 per month?

44. Find the difference between the sum of 6789, 65,432, 4586, and the sum of 7689, 4532, 4326.

45. Find the cost of 8756 acres of land at $ 67 per acre.

46. From the sum of $ 67.84, $ 78.15, $ 45.12, and $ 6.85, take the sum of $ 13.45, $ 26.15, $ 12.50, $ 32.25.

47. Divide 678,943 by 67.

48. The product of two numbers is 720. One of the factors is 80; what is the other factor?

49. The sum of two numbers is 78,693. One of the numbers is 45,763; what is the other number?

50. A farmer gathers an average of 45 bushels of oats per acre from a field of 87 acres. How many bushels does he gather from the entire field?

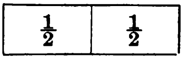
59. When an apple is cut into 2 *Equal* parts, we do not call one of the parts a piece, but call it one half. We cannot call it one half unless the parts are *Equal*.

One half is written $\frac{1}{2}$.

Fold a paper, and show me $\frac{1}{2}$ of it. How would you cut an orange to give me $\frac{1}{2}$ of it? What part of the orange would you have left, after giving me $\frac{1}{2}$ of it?

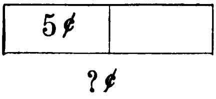
$\frac{1}{2}$ means one of the two equal parts of something.

Fold a small piece of paper, and crease it to show halves.



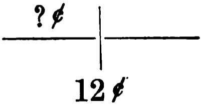
Use this to help you work the following:

1. If $\frac{1}{2}$ yard of ribbon costs 5¢. what will 1 yard cost?

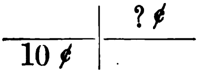


10¢ *Ans.*

2. If 1 yard ribbon costs 12 ¢, what will ½ yard cost?



3. ½ pound candy costs 10 ¢; what will 1 pound cost?



4. 1 pound candy costs 18 ¢; what will ½ pound cost?

Make a drawing to show your work. You may use any kind of drawing that shows plainly what you understand the question to be and how you will answer it.

5. If a bushel of potatoes costs $1, what will one half bushel cost?

6. If ½ lb. tea costs 25 ¢, what will 1 lb. cost?

7. If a basket of peaches costs 80 ¢, what will ½ basket cost?

8. A boy had 10 marbles and lost ½ of them. How many did he lose? How many had he left?

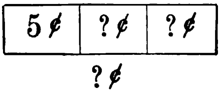
9. A boy had 20 ¢ and spent ½ of it. How much did he spend?

10. If 10 ¢ will buy ½ pound of candy, how much money will be needed to buy one pound?

60. When anything is divided into 3 *Equal* parts, each part is called one third. Written $\frac{1}{3}$.

Fold a paper to show thirds. Use it to help you work the following:

1. $\frac{1}{3}$ lb. candy costs 5 ¢; what will 1 lb. cost?



2. If I had 9 marbles and lost $\frac{1}{3}$ of them, how many did I lose? How many marbles left?

3. If I cut a watermelon into 3 equal parts and give you one of the parts, what part of the whole melon would you receive?

4. If $\frac{1}{3}$ costs 5 ¢, what will $\frac{2}{3}$ cost?

5. If I sell $\frac{1}{3}$ of my farm, what part of the farm will I have left?

6. A man spends $\frac{2}{3}$ of his money. What part of his money has he kept?

7. A man has $6 and spends $\frac{1}{3}$ of it for shoes. How much money did he spend? How much money has he left? What part of the $6 has he left?

8. A boy buys some peanuts. After eating $\frac{1}{3}$ of them, he gives me the remainder. What part of the peanuts which he bought do I get?

9. I have 15 marbles. I give James $\frac{1}{3}$ of them and give you $\frac{1}{3}$ of them. Which has the

most marbles, James, you, or I? How many will I have?

10. There are 24 apples in a box. If I take out $\frac{1}{3}$ of them and give them to you, how many will you receive? How many will be left in the box? What part of the box of apples will be left in the box?

61. $\frac{1}{4}$ means one of the four equal parts of a unit.

$\frac{3}{4}$ means three of the four equal parts of a unit.

Look at this picture and see if you can tell me why we do not use $\frac{2}{4}$.

$$\begin{array}{c|c|c|c} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \hline \multicolumn{2}{c|}{\frac{1}{2}} & \multicolumn{2}{c}{\frac{1}{2}} \end{array}$$

Fold paper so as to show you fourths, and make drawings to help you, and to show that you understand the work. Try to do the following examples without any further help:

1. $\frac{1}{4}$ of a pound of candy costs 5 ¢. What will 1 pound cost?

2. If 1 lb. of candy costs 24 ¢, what will $\frac{1}{4}$ lb. cost?

3. If 1 lb. of candy costs 36 ¢, what will $\frac{1}{4}$ lb. cost? What will $\frac{3}{4}$ lb. cost?

4. If I cut a pie in 4 equal pieces, give you one piece, and take one myself, how many pieces

will remain? What part of the pie will remain? (Make a picture.)

5. A boy had 16 marbles and lost $\frac{1}{4}$ of them. How many did he lose?

6. If a quarter of a pound of tea costs 10 ¢, what will one half pound cost?

7. If $\frac{1}{2}$ lb. of coffee costs 16 ¢, what will $\frac{1}{4}$ lb. cost?

8. How many more apples will I get if I take $\frac{1}{2}$ of a pile of 20 apples than if I take $\frac{1}{4}$ of the pile?

9. John had 12 ¢. He spent $\frac{1}{3}$ of it for an orange. How much had he left? Could he buy five cents' worth of peanuts with $\frac{1}{2}$ of what he has left?

10. Find the cost of $\frac{1}{2}$ lb. tea at 50 ¢ a pound, $\frac{1}{3}$ lb. cheese at 15 ¢ a pound, and $\frac{3}{4}$ lb. butter at 24 ¢ a pound.

FRACTIONS

62. In the fraction $\frac{2}{3}$, which number tells us into how many parts the unit is divided? Which tells us how many parts are used?

In the fraction $\frac{3}{4}$, which number tells us into how many parts the unit is divided? Which number tells us how many parts are used?

63. To be remembered.

1. A fraction is one or more of the equal parts of a unit.

2. The number written below the line in a fraction tells us into how many parts the unit is divided. It is called the **denominator**. It shows the name of the parts.

3. The number written above the line in a fraction tells us how many of the parts of the unit are used. It is called the **numerator**. It shows the number of the parts used.

64. To find $\frac{1}{2}$ of a number, we divide it into 2 equal parts.

To find $\frac{1}{3}$ of a number, we divide it into 3 equal parts.

To find $\frac{1}{4}$ of a number, we divide it into 4 equal parts.

To find $\frac{1}{5}$ of a number, we divide it into 5 equal parts.

How can we find $\frac{1}{6}$ of a number? $\frac{1}{7}$ of a number? $\frac{1}{8}$ of a number? How can we find $\frac{1}{9}$ of an orange? How can we find $\frac{1}{4}$ of an apple? How can we find $\frac{1}{5}$ of a melon?

How can we find $\frac{1}{4}$ of **any** unit?
How can we find $\frac{1}{5}$ of **any** unit?
How can we find $\frac{1}{6}$ of anything?
How can we find $\frac{1}{9}$ of anything?

65. To find $\frac{2}{3}$ of anything, we first divide it into the number of parts shown by the denominator, 3, and then take the number of those parts which the numerator shows, 2.

How can you find $\frac{3}{4}$ of a number?
How can you find $\frac{4}{5}$ of a number?
How can you find $\frac{7}{8}$ of anything?
How can you find $\frac{3}{7}$ of an orange?

66. Find the parts of numbers shown in these examples.

1. $\frac{1}{2}$ of 28.
2. $\frac{2}{3}$ of 12.
3. $\frac{3}{4}$ of 20.
4. $\frac{2}{3}$ of $6.
5. $\frac{3}{4}$ of 8 lb.
6. $\frac{4}{5}$ of $25.
7. $\frac{3}{8}$ of $240.
8. $\frac{1}{3}$ of 369.
9. $\frac{2}{3}$ of 18 bushels.
10. $\frac{1}{5}$ of 10 acres.
11. $\frac{3}{4}$ of $36.
12. $\frac{4}{5}$ of $15.
13. $\frac{1}{8}$ of 640.
14. $\frac{2}{3}$ of $24.
15. $\frac{3}{8}$ of $16.
16. $\frac{5}{8}$ of $49.
17. $\frac{3}{5}$ of 10¢.
18. $\frac{7}{8}$ of 24¢.
19. $\frac{3}{7}$ of 21 apples.
20. $\frac{7}{8}$ of 16 marbles.

67. Make drawings to show what each of the following fractions mean. Do not make all the drawings in the same form, but use circles for some, squares for others, lines, etc.

1. Show what is meant by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$.
2. Show what is meant by $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

3. Show what is meant by $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$.

4. Show what is meant by $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$.

68. Written Exercises

1. What will $\frac{1}{3}$ of a $2400 farm cost?

2. What will $\frac{2}{3}$ of a yard of ribbon cost at 24¢ per yard?

1 yd. costs 24¢,
$\frac{1}{3}$ yd. costs ? ¢,
$\frac{2}{3}$ yd. costs ? ¢. *Ans.*

3. What will $\frac{3}{4}$ lb. of tea cost at 48¢ a pound?

4. Find the cost of $\frac{3}{8}$ lb. of butter at 32¢ a pound.

5. A man sold $\frac{1}{2}$ of his farm for $1500. What was the remainder worth at the same rate?

6. A boy lost $\frac{2}{3}$ of his marbles and had 6 marbles left. How many marbles had he at first? (Use drawing or folded paper, if necessary, to see how to work this example.)

7. After spending $\frac{3}{4}$ of my money, I had $5 left. How much money had I at first? (Make drawing.)

8. $\frac{1}{8}$ of a pound of tea costs 5¢. What will a whole pound cost at the same rate?

9. If $\frac{1}{8}$ lb. of tea costs 8¢, how many pounds can you buy for $1.28?

10. At $\frac{1}{4}$ of a dollar a pound, how much candy can I buy for $2?

MIXED NUMBERS

69. $1\frac{1}{2}$ means one whole unit and $\frac{1}{2}$ of another.
$4\frac{2}{3}$ means 4 whole units and $\frac{2}{3}$ of another.
$5\frac{3}{8}$ means 5 whole units and $\frac{3}{8}$ of another.
$4\frac{3}{8}$ apples means 4 apples and $\frac{3}{8}$ of another apple.

70. When fractions and whole numbers are written together in this way, they are called mixed numbers.

We can use mixed numbers as either factor in multiplication, if we multiply the whole number and the fraction separately by the other factor, and then add the products.

A little later we shall learn how to use mixed numbers for both factors. Most examples in business, however, will not use more than one factor as a mixed number.

71. Written exercises using one factor as a mixed number.

1. Find the cost of $3\frac{1}{2}$ lb. of cheese at 16¢ a pound.

16¢	Multiplying 16 by $\frac{1}{2}$, we have 8.
$\times 3\frac{1}{2}$	Multiplying 16 by 3, we have 48.
8	Adding the products, we have 56.
48	We then see what kind of units the
56¢ *Ans.*	answer shows, and write 56¢.

The plan of keeping the work abstract, and then deciding what kind of units the answer shows, saves much trouble about the use of the multiplier as abstract.

2. Find the cost of $35\frac{1}{2}$ lb. of butter at 32¢ a pound.

3. Find the cost of 16 lb. of butter at $36\frac{1}{2}$¢ a pound.

$$\begin{array}{r} 36\frac{1}{2}\text{¢} \\ \times 16 \\ \hline 8 \\ 216 \\ 36 \\ \hline \end{array}$$

584¢, or $5.84 *Ans.*

You can think 16 times $\frac{1}{2}$ are 8, or $\frac{1}{2}$ of 16 is 8, in multiplying the fraction by $\frac{1}{2}$ by 16.

4. Find the cost of 38 yd. of cloth at $8\frac{1}{2}$¢ a yard.

$$\begin{array}{r} 38 \\ \times 8\frac{1}{2} \\ \hline 19 \\ 304 \\ \hline 323 \end{array}$$

It will cost 38 times $8\frac{1}{2}$¢, and the answer will be in cents, 38 times $8\frac{1}{2}$ is 323¢, or $3.23. *Ans.*

It is better for this class of examples to write them in the form —

$$8\tfrac{1}{2}\text{¢} \times 38 = \$\,?$$

and then do the work of multiplication in the abstract form to find the number of dollars.

5. Find the cost of 36 lb. of sugar at $5\frac{1}{2}$¢ a pound.

$$5\tfrac{1}{2}\text{¢} \times 36 = \$\,?$$

6. What is the cost of 60 lb. of butter at 32½¢ a pound?

7. What is the cost of 12½ acres of land at $60 an acre?

8. What is the cost of 36¼ yd. of cloth at 12¢ a yard?

9. Bought 4½ lb. of rice at 12¢ a pound, and gave the clerk $1 in payment. How much change should I receive?

10. Find the total cost of the following bill of goods:

12½ lb. sugar at 6¢,
¼ lb. tea at 48¢,
1½ bushels potatoes at 90¢,
16 lb. butter at 32½¢.

11. Find the cost of 240 lb. sugar at 5½¢ a pound.

12. Find the cost of 160 bushels of oats at 25½¢ a bushel.

13. What will 48 barrels of apples cost at $3½ a barrel?

14. What will 90 bushels of wheat cost at 85½¢ a bushel?

15. What will 36¼ yd. of velvet cost at $2 a yard?

16. Find the cost of 6⅜ lb. of cheese at 16¢ a pound.

17. What will $35\frac{1}{2}$ lb. of butter cost at 24¢ a pound?

18. Find the cost of $7\frac{1}{2}$ gallons of oil at $12\frac{1}{2}$¢ a gallon.

19. A man bought $16\frac{1}{4}$ acres of land at \$80 per acre. What did it cost him?

20. Bought $\frac{3}{4}$ yd. of cloth at 80¢ a yard and $1\frac{1}{2}$ yards at 60¢ a yard. What did both cost?

21. What will $\frac{4}{5}$ of an acre of land cost at \$120 per acre?

22. What will $165\frac{1}{2}$ bushels of potatoes cost at 50¢ a bushel?

23. Find the cost of three quarters of a pound of tea at 80¢ a pound. (Fourths are often called quarters.)

24. What will $\frac{3}{4}$ bushels of chestnuts cost at \$1.20 a bushel?

72. Drill applications.

1. Cost of $15\frac{1}{2}$ acres of land @ \$45 per acre?
2. Cost of 252 pounds of beef @ $9\frac{1}{4}$¢ a pound?
3. Cost of $18\frac{2}{3}$ barrels of flour @ \$7.60 a barrel?
4. Cost of 15 acres of land @ \$$45\frac{2}{3}$ per acre?
5. Cost of $18\frac{3}{4}$ pounds of coffee @ 24¢ a pound?

In the following exercises, the name of the article is not given, that the pupils may supply suitable names as a matter of interest.

6. 27 @ 15$\frac{1}{3}$¢?

7. 45$\frac{1}{2}$ @ 80¢?

8. 8$\frac{3}{4}$ @ 16¢?

9. 36 @ 12$\frac{1}{2}$¢?

10. 24$\frac{1}{2}$ @ 6¢?

11. 8$\frac{1}{3}$ @ 27¢?

12. 150 @ 4$\frac{1}{2}$¢?

13. 48 @ 32$\frac{1}{2}$¢?

14. 64 @ 5$\frac{1}{8}$¢?

15. 45 @ 4$\frac{2}{5}$¢?

16. 340$\frac{1}{2}$ @ $48?

17. 980 @ 12$\frac{1}{2}$¢?

18. 74 @ 7$\frac{1}{2}$¢?

19. 380 @ 2$\frac{4}{5}$¢?

20. 6780 @ 35\frac{1}{2}$?

Note to the Teacher. Statements are often made that city pupils cannot work examples when the commodity has a name not familiar to them. It is said that they cannot find the amount of butter fat in a given quantity of milk, the amount of fat per pound being given, etc. It is also said that pupils in the country sections have a similar difficulty in working exercises using terms strange to them. Exercises where the commodity is not named, but is left for the pupil to fill, or the answer found without consideration of *any* commodity, should remedy the trouble named. It also enables the teacher to give a large amount of drill work by a little writing on the board, and is more interesting than the abstract drill work, where the pupil is told to multiply the following numbers, and then lines of figures are given.

73. Find the cost of whole units when the cost of fractional parts are given.

1. If $\frac{1}{4}$ lb. of candy costs 25¢, what will 1 pound cost?

$\frac{1}{4}$ costs 25¢ $\frac{4}{4}$ cost ?¢

2. If $\frac{3}{4}$ lb. of candy cost 15¢, what will 2 lb. cost?

$\frac{3}{4}$ cost 15¢
$\frac{1}{4}$ costs 5¢
$\frac{4}{4}$ cost 20¢
2 cost 40 *Ans.*

Avoid the oral analysis which says, If $\frac{3}{4}$ cost 15¢, $\frac{1}{4}$ will cost $\frac{1}{3}$, etc., as $\frac{1}{4}$ costing $\frac{1}{3}$ is often confusing.

For oral analysis use, $\frac{3}{4}$ cost 15¢, $\frac{1}{4}$ costs 5¢, $\frac{4}{4}$ or 1 lb. costs 20¢, 2 lb. cost 40¢. *Ans.*

3. What will 6 lb. of coffee cost if I pay at the rate of 15¢ for $\frac{1}{2}$ lb.?

4. Find the cost of 12 bushels of chestnuts at the rate of $\frac{3}{4}$ bushel for 60¢.

5. If $\frac{3}{8}$ lb. of cheese cost 6¢, what will 3 pounds cost at the same rate?

6. What will 2 melons cost if $\frac{1}{4}$ melon costs 8¢?

7. What will $\frac{2}{3}$ of a twenty-five cent melon cost?

COMMON MEASURES

Note. For convenience of reference, these tables are all printed on one page. They need not all be learned at the same time.

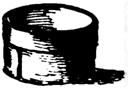
74.

LIQUID MEASURE

2 pints make 1 quart,
4 quarts make 1 gallon.

Abbreviations: pt. for pint, qt. for quart, gal. for gallon.

Pint.



Quart.

Peck.

Bushel.

75.

DRY MEASURE

2 pints make 1 quart,
8 quarts make 1 peck,
4 pecks make 1 bushel.

Abbreviations: pt. for pint, qt. for quart, pk. for peck, bu. for bushel.

76.

MEASURES OF LENGTH

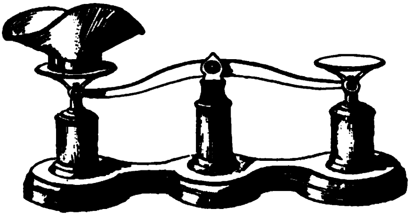
12 inches make 1 foot,
3 feet make 1 yard,
1760 yards or 5280 feet make 1 mile.

Abbreviations: in. for inch, ft. for foot, yd. for yard, mi. for mile.

77. MEASURES OF WEIGHT

16 ounces make 1 pound,
2000 pounds make 1 ton.

Abbreviations: oz. for ounces, lb. for pound, T. for ton.



78. EXERCISES IN LIQUID MEASURES

1. How many pints in 2 qt.? in 4 qt.? in 6 qt.?

2. How many quarts in 4 pt.? in 12 pt.? in 16 pt.?

3. Find the cost of 2 qt. of milk at 3¢ a pint.

4. Find the cost of 1 pt. of milk at 10¢ a quart.

5. Find the cost of 2 qt. at 20¢ a gallon.

6. Find the cost of 1 gal. at 8¢ a quart.

7. What will $\frac{1}{2}$ gal. of oil cost at 8¢ a quart?

8. What will $1\frac{1}{2}$ qt. of milk cost at 5¢ per pint?

9. Find the cost of $\frac{3}{4}$ gal. at 12¢ per quart.

10. Bought 3 gal. oil at 40¢ per gal., and sold it at 15¢ a quart. How much did I gain?

EXERCISES IN DRY MEASURES

Note. Pints and quarts used in measuring liquids are a little smaller than the pints and quarts used in dry measure.

79. 1. Bought potatoes at $1 per bushel, and sold them at 30¢ per peck. How much did I make on each bushel?

2. Bought 1 pk. of peanuts for $.50, and sold them at 10¢ a quart. How much did I gain?

3. Potatoes are selling at 80¢ a bushel. How much should I pay for 3 pk.?

4. When chestnuts are 5¢ a quart, how much should I pay for 2 pk. of chestnuts?

5. Find the total cost,

3 pk. oats at 40¢ a bushel,
2 qt. popcorn at 36¢ a peck.

6. How many quarts are there in 1 bu.? in $\frac{1}{2}$ bu.?

7. Find the cost of $\frac{1}{2}$ bu. berries at 10¢ a quart.

8. If I feed a horse 3 times each day, and give him 4 qt. of oats each time, how many bushels will I feed him in 3 days?

9. I buy apples at $1.20 a bushel, and sell them at 40¢ a pk. How much do I make on each bushel?

10. How many quarts in $4\frac{1}{2}$ bu.?

EXERCISES IN MEASURES OF LENGTH

80. 1. How many feet in 17 yd.?

2. How many yards in 36 ft.?

3. How many inches in 8 ft.?

4. How many feet in 144 in.?

5. How many yards in ½ mi.?

6. How many feet in ¼ mi.?

7. What will it cost to build a fence ¼ mi. long at 50¢ a yard?

8. What will 45 ft. of cloth cost at 25¢ a yard?

9. How many inches in 1 yd.? in ¼ yd.?

10. It is 285 ft. around a lot. What will it cost to build a fence around it at $1 a yard?

81. EXERCISES IN MEASURES OF WEIGHT

1. What will 8 oz. of tea cost at 50¢ a pound?

2. At 5¢ an ounce what will 1 lb. of tobacco cost?

3. How many pounds in ½ ton? in ¼ of a ton?

4. I buy meal at $12 per ton, and sell it at 1¢ a pound. How much do I make on each ton?

5. How many pounds in 3½ tons of coal?

6. How many pounds in 2¼ tons of hay?

7. Find the cost of 12 oz. of cheese at 16¢ a pound.

8. Find the cost of 2 lb. nutmeg at 3¢ an ounce.

9. Find the cost of 360 lb. of potatoes at 80¢ a bushel. (60 lb. to the bushel for potatoes.)

10. Find the cost of 1260 lb. of wheat at 95¢ a bushel. (60 lb. to a bushel for wheat.)

TABLE OF WEIGHTS OF A BUSHEL. (For reference.)

Barley, 48 lb. to the bushel.
Corn (shelled), 56 lb. to the bushel.
Oats, 32 lb. to the bushel.
Potatoes, 60 lb. to the bushel.
Rye, 56 lb. to the bushel.
Wheat, 60 lb. to the bushel.

82. GENERAL REVIEW

1. How many gallons in 320 pints?

2. A man picked $3\frac{1}{2}$ bu. of cherries in quart boxes. How many boxes did he use?

3. A farmer feeds his horses 8 qt. of oats each day. How many bushels will he use in 8 days?

4. An empty freight car weighs 15 tons. Loaded with wheat, it weighs 33 tons. How many pounds of wheat does it contain? How many bushels? (Wheat weighs 60 lb. a bushel.)

5. A grocer sells a sack of coffee weighing 125 lb. for $18.75. What does he receive per pound for the coffee?

6. Find the total cost of the following-named groceries: 12 lb. butter at 25¢ a pound; 6 dozen eggs at 30¢ a dozen; $5\frac{1}{2}$ lb. coffee at 30¢ a pound;

$\frac{1}{2}$ lb. tea at 80¢ a pound; 12 qt. cranberries at $12\frac{1}{2}$¢ a quart.

7. If 25 lb. coffee cost $ 9, what will 6 lb. cost?

8. A boy bought $\frac{1}{2}$ lb. candy, and gave away $\frac{1}{4}$ lb. What part of a pound had he left?

9. Find the cost of $\frac{5}{8}$ yd. of silk at 80¢ a yard.

10. If half a dozen lemons cost 15¢, what will 4 dozen cost?

11. What will $2\frac{3}{4}$ lb. of mixed candy cost at 20¢ a pound?

12. At 9¢ a yard, what will $8\frac{1}{3}$ yd. of cloth cost?

13. Find the cost of $\frac{5}{8}$ yd. of velvet at $ 2.40 per yard.

14. What is the cost of 16 lb. cheese at $14\frac{1}{2}$¢ a pound?

15. Find the cost of 48 yd. cloth at $33\frac{1}{3}$¢ a yard.

16. If I pay $ 240 a year rent for a house, what is the rent for 5 months at the same rate?

17. I buy 60 sheep for $ 240, and sell them at $ 4.50 each. What do I make on each sheep? On the whole number sold?

18. Paid $ 8000 for a farm of 160 acres, and sold it at $ 60 per acre. How much did I gain?

19. At the rate of 3 oranges for 10¢, how many oranges can I buy for half a dollar?

20. At the rate of 2 apples for 5¢, what will 24 apples cost?

21. If 3 lb. of rice cost 22¢, what will 12 lb. cost?

22. If 6 lb. of sugar cost 25¢, how many pounds can I buy for one dollar?

23. When cotton costs $10\frac{5}{8}$¢ a pound, what is the cost of 2 bales of 500 lb. each?

24. If $\frac{1}{2}$ lb. of bacon cost 5¢, what will 4 lb. cost at the same rate?

25. What is the weight of $2\frac{3}{4}$ bushels of corn, allowing 56 lb. to the bushel?

CHAPTER II

PLACE VALUE OF FIGURES

83. 1. Compare the values of \$.05; \$.50; \$5.00; \$50.; \$500.; \$5000.

Moving the 5 each place to the left multiplied its value how many times?

2. Compare the values of \$3000; \$300; \$30; \$3; \$.30; \$.03.

Moving the 3 each place to the right divided its value by what number?

3. Compare the values of 9000; 900; 90; 9.

4. Compare the values of 6; 60; 600; 6000; 60,000.

Moving a figure one place to the *left* multiplies its value by 10.

Moving a figure one place to the *right* divides its value by 10.

Moving a figure two places to the left multiplies its value by what? Moving a figure two places to the right divides its value by what?

84. To be remembered.

1. The value of a figure depends upon its place in the number.

2. Its value at the *left* of units' place is 10 times its unit value in the first place, 100 times its unit value in the second place, 1000 times its unit value in the third place, etc.

3. Its value at the *right* of units' place is $\frac{1}{10}$ its unit value for the first place, $\frac{1}{100}$ of its unit value for the second place, $\frac{1}{1000}$ of its unit value for the third place, etc.

4. The decimal point is used to show us where the whole number ends and the tenths of the number begin. In $ 125.45, the point after the 5 shows us that the whole dollars end there. The next figures after the decimal point are parts of dollars, the first figure being $\frac{4}{10}$ dollar, and the second figure being $\frac{5}{100}$ dollar in the number used.

85. Exercises in place value of figures.

1. How many times is 3 divided by 10 to make .3? .03? .003?

2. 6 is multiplied by ten when we change 6 to 60. Is it multiplied or divided when we change .006 to .06? When we change .07 to .7? When we change .7 to 7.?

3. Tell what is done when the numbers are changed as shown below:

600. changed to 60.
.07 changed to 7.
40. changed to .04.
.06 changed to .006.
.03 changed to 30.
5. changed to .005.

400. changed to 4.
.08 changed to 80.
8. changed to .008.
400. changed to .4.
70. changed to .07.
60. changed to .6.

4. In the number 45.15, I move the decimal point to the next place, and it is 451.5. Tell me what change is made in the value by this change in the position of the decimal point.

5. $456. $4.56 $45.60

Can you see what change was made? Are the values changed?

86. Chart showing the Place Value of Figures in Integers and Decimals.

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	UNITS	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths]	Millionths
6	**5**	**4**	**3**	**2**	**1**	**X.**	**1**	**2**	**3**	**4**	**5**	**6**

If we consider units' place the KEY, and remember that the decimal point is used *after* units to separate the integers from the decimals, we can count from *units'* place toward the right or the left and find similar names for the place value of the figures at corresponding distances from units' place.

The decimal parts have the ending *ths*, tens and tenths, hundreds and hundredths, thousands and thousandths, being integral and decimal names for the place value of the figures one, two, and three places to the left and the right of *units'* place.

CAUTION. Do not start with the decimal point to numerate numbers having decimal parts, but use the point to locate units, and start with units, that the association of integral and decimal notation may be noted.

If we begin at units' place, we can read the numbers at the right of the decimal point as easily as we have learned to read them at the left. The comma may be used for large numbers to help us group the figures. In reading parts of numbers at the right of the decimal point, we read them as whole numbers, and *then* give them the place name of the right-hand or last figure.

.01234 is read one thousand, two hundred thirty-four *hundred thousandths*, as it ends in hundred thousandths' place.

In reading numbers like 245.245, we read the decimal point as "and," so any one hearing us and not seeing the number may know where to place the decimal point. 245.245 is read, 245 and 245 thousandths. Do not forget to use the *ths* at the end of the decimal parts.

Read the following numbers:

1. 34.056 345.0065 65.075 .0543 8.0005.
2. 125.006 58.004 90.0085 6.0006 8.008.

READING AND WRITING DECIMALS

We seldom use decimals beyond thousandths, but we should learn to read and to write them to thousandths, so that we can use them easily and rapidly.

87. Read the following decimals:

1. 15.02. Read, fifteen and two hundredths.
2. 8.07, 9.15, 12.18, 14.25, 16.85, 75.145.
3. .015, .185, 7.125, 8.006, 6.140, 27.302.
4. 32.032, 87.87, 87.087, 870.087, 870.87, .087.
5. 9.109, 7.246, 3.065, 48.074, 850.409, 50.056.
6. 1.234, 2456.637, 4537.796, 6706.005, 6783.306.
7. 1050.304, 1253.009, 1453.238, 8090.457, 78.078.
8. 5658.008, 4368.258, 6853.321, 7650.352, 9.009.
9. 975.004, 865.231, 650.405, 684.312, 75.075.
10. 34567.38, 36409.045, 875352.005, 650408.007.

WRITING DECIMALS

(Decimals may be written from dictation exercises, as the form written out in words is of no value to pupils at this age, and of little value in after life except in its limited use in writing in the words for value in checks, etc.)

ADDITION OF DECIMALS

88. We add decimals as we have learned to add dollars and cents. Write the numbers so that the decimal points shall stand directly under each other, and then add and place the point in the answer directly under the column of points. The right-hand figure of the whole numbers will be straight up and down, as it is next to the decimal points.

Add:

$$\begin{array}{r@{}l}
569 & .008 \\
35 & .469 \\
145 & . \\
 & .065 \\
32 & .543 \\
3 & .27 \\
145 & .5 \\
\hline
930 & .855 \quad \textit{Ans.}
\end{array}$$

89. Add each line of decimals that is numbered separately in the exercise given for "Reading Decimals," on the opposite page.

90. Read the following decimals, arrange them for addition, and then find their sum:

1. 540.785, 678.352, 76543.25, 6547.007.
2. 3467.235, 56345.007, 6783.063, 456.378.
3. 987.357, 467895.356, 45685.25, 46783.351.
4. 468.345, 456321.357, 4569.364, 6845.306.
5. 465321.453, 400074.432, 456.006, .008.

SUBTRACTION OF DECIMALS

91. Subtraction of decimals is like subtraction of simple numbers, except that we have to be careful to write the numbers so that the decimal points shall stand directly under each other.

$$\begin{array}{r} 67895 \\ -\ 35432 \\ \hline 32463 \end{array} \qquad \begin{array}{r} 678.95 \\ -\ 354.32 \\ \hline 324.63 \end{array}$$

The only trouble you will be likely to have in subtraction of decimals is in exercises like this: Subtract 3.125 from 345.

$$\begin{array}{r} 345.000 \\ -\ 3.125 \\ \hline \end{array}$$

In writing the upper number, the minuend, we have to fill out the places with ciphers, if there are more decimal places in the lower number (the subtrahend) than are written in the minuend. Ciphers at the end and at the right of units' place make no difference in the value of the number. $ 6. is the same as $ 6.00, as you have already learned. 67.15 is the same as 67.150000 in value.

92. 1. From two hundred subtract two hundredths.

2. From 1000 subtract .003.

3. From $875. subtract 75¢.

4. From the sum of 67.892 and 34.523 subtract the sum of 67.652 and 12.345.

5. From 35 subtract .105.

6. From one thousand subtract one thousandth.

7. In a bin that will hold 270.5 bu. of wheat there are 129,025 bu. How many more bushels will it hold?

8. From five millions subtract five millionths.

9. From .875 acres of land five tenths acres were sold. What decimal part of an acre remained? Write the answer in words.

MULTIPLICATION OF DECIMALS

93. Multiplication of decimals is the same as multiplication of whole numbers; the placing of the decimal point to locate units' place in the product is the only point requiring especial attention.

350	$3.50	.350	350	350	3.50
× 4	× 4	× .4	× 4	× .04	× .4
1400	$14.00	1.400	140.0	14.00	1.400

If you will carefully study the pointing off of the product in each of the above examples, you will see that there are as many places pointed off in the product as there are decimal places in both multiplier and multiplicand.

When we multiply tenths by tenths, the product is hundredths. When we multiply hundreds by tenths, the product is thousandths.

To multiply where there are decimal places in one or both of the factors, we multiply as in whole numbers, and point off as many decimal places in the product as there are decimal places in *both* factors.

94. 1. Multiply 875 by .05. Multiply 67.5 by .03.

2. 67.05 × .8. .008 × 345. 894 × 025.

3. 8.75 × 234.5. 789.6 × 67.15. 453.25 × .7.

4. Find the cost of 15.7 cords of wood at $2.40 a cord.

DIVISION OF DECIMALS

95. Division of decimals is like division of whole numbers, except that the matter of locating units' place in the quotient requires careful attention.

$$\begin{array}{cccc}
\begin{array}{r} 342 \\ 2\overline{)684} \end{array} & \begin{array}{r} 34.2 \\ 2\overline{)68.4} \end{array} & \begin{array}{r} 3.42 \\ 2\overline{)6.84} \end{array} & \begin{array}{r} .342 \\ 2\overline{).684} \end{array} \\
\begin{array}{r} 34200 \\ .02\overline{)684.} \end{array} & \begin{array}{r} 3420 \\ .02\overline{)68.4} \end{array} & \begin{array}{r} 342 \\ .02\overline{)6.84} \end{array} & \begin{array}{r} 34.2 \\ .02\overline{).684} \end{array} \\
\begin{array}{r} 342000 \\ .002\overline{)684.} \end{array} & \begin{array}{r} 34200 \\ .002\overline{)68.4} \end{array} & \begin{array}{r} 3420 \\ .002\overline{)6.84} \end{array} & \begin{array}{r} 342 \\ .002\overline{).684} \end{array}
\end{array}$$

If we study the above examples, and prove any one of them by multiplying the divisor and quotient together to see if the product is equal to the dividend, we shall see much that is like what we have already learned.

There are two methods of locating the decimal point in the quotient. You do not need to remember both methods, but you should select the method which you understand and can use without making mistakes. Some use one method, some the other, so both are given below.

First Method

1. If there are more decimal places in the divisor than in the dividend, annex ciphers to the dividend until the decimal places equal those of

the divisor in number. The answer will be a whole number.

2. If the number of decimal places in the divisor is less than those of the dividend, point off enough places in the quotient to make the sum of decimal places in the divisor and quotient equal to the decimal places in the dividend.

Note. This method is based upon the facts learned in multiplication, and the fact that the divisor and quotient when multiplied together must produce the dividend.

Second Method

1. If the divisor is a whole number, divide as we have learned to do in dollars and cents, and place the point in the quotient directly over the point in the dividend.

2. If the divisor has decimal places, multiply both divisor and dividend by moving the decimal point the same number of places to the right in both, until the divisor becomes a whole number. Then divide as in the first step just given.

Example. Divide 93.6 by .003.

First method	Proof
$\begin{array}{r} 31200 \\ .003\overline{)93.600} \end{array}$	$\begin{array}{r} 31200 \\ \times .003 \\ \hline 93.600 \end{array}$

Second method	Proof
$\begin{array}{r} 31\,200 \\ .003\overline{)93.600} \end{array}$	$\begin{array}{r} 31200 \\ \times .003 \\ \hline 93.600 \end{array}$

96. Write the answers at sight:

1. 9 ÷ 3 = ?   8 ÷ 2 = ?   .12 ÷ 4 = ?
2. .09 ÷ 3 = ?   .008 ÷ 2 = ?   .012 ÷ 4 = ?
3. 9 ÷ .03 = ?   8 ÷ .02 = ?   12 ÷ .004 = ?
4. 12 ÷ .003 = ?   .08 ÷ .02 = ?   .008 ÷ .004 = ?
5. 25 ÷ .005 = ?   .25 ÷ .005 = ?   .012 ÷ .04 = ?
6. .25 ÷ .005 = ?   .025 ÷ .005 = ?   120 ÷ .04 = ?
7. .81 ÷ .09 = ?   56 ÷ .08 = ?   24 ÷ .004 = ?
8. 81 ÷ .09 = ?   5.6 ÷ .08 = ?   2.4 ÷ .004 = ?
9. .081 ÷ .009 = ?   .056 ÷ .008 = ?   .024 ÷ .04 = ?
10. .81 ÷ .009 = ?   .56 ÷ .008 = ?   .024 ÷ .004 = ?

97. Carrying out to change remainder to a decimal.

1. If we divide 9 by 4, we have an answer of 2$\frac{1}{4}$.

$$\begin{array}{r} 2\frac{1}{4} \\ 4\overline{)9} \end{array} \qquad\qquad \begin{array}{r} 2.25 \\ 4\overline{)9.00} \end{array}$$

We can annex ciphers and carry on the division until there is no remainder, and have our answer in a different form.

Carry out the remainders, and write answers in the decimal form:

1. 1 ÷ 8   1 ÷ 4   12 ÷ 16
2. 3 ÷ 8   3 ÷ 4   12 ÷ 32
3. 7 ÷ 8   1 ÷ 40   28 ÷ 32
4. 4 ÷ 16   3 ÷ 40   15 ÷ 20
5. 6 ÷ 16   18 ÷ 24   7 ÷ 56

6. 14 ÷ 16   27 ÷ 36   21 ÷ 56
7. 27 ÷ 72   24 ÷ 32   9 ÷ 72
8. 21 ÷ 28   9 ÷ 360   24 ÷ 320
9. .3 ÷ .8   .1 ÷ .4   12 ÷ 1.6
10. .003 ÷ .008   .001 ÷ .004   .012 ÷ .016

DIVISION OF DECIMALS

98. 1. Divide 129.591 by .63.
2. Divide 69.0624 by 3.6.
3. Divide 20.703 by 3.09.
4. Divide 26.740 by .764.
5. Divide .12816 by 3.56.
6. Divide 45.0144 by 3.6.
7. Divide .36323 by .5189.
8. Divide 22.526 by 3.218.
9. Divide 45.664 by 57.08.

99. If the division is not exact in whole numbers, given below, carry out the quotient to secure the decimal form to the remainder.

1. 38 ÷ 152.   2. 16 ÷ 128.   3. 450 ÷ 64.
4. 38642 ÷ 124.   5. 73928 ÷ 268.   6. 86935 ÷ 325.

7. If a barrel of flour costs \$5.75, how many barrels can be bought for \$285.75?

8. There are 31.5 gallons in a barrel. How many barrels of water are there in 2787.75 gallons?

9. If a man can travel 33.68 miles in 8 days, how far can he travel in 7.5 days?

10. A farm of 71.5 acres cost me $6220.50. What was the cost per acre?

11. Divide .04905 by .327.

12. Divide 725.406 by .0957.

13. If a man earns $162 in 13.5 weeks, what are his average wages per week?

14. If 38.5 bales of cloth cost $3048.45, what is the price of each bale?

15. At $.045 per pound, how many pounds of sugar can be purchased for $1.89?

16. At $2.625 a yard, how many yards of cloth can be purchased for $61.005?

17. At .47 of a dollar a bushel, how many bushels of corn can be purchased for $222.545?

18. If a man travels 29.6 miles in a day, how many days will it take him to travel 1016.088 miles?

19. A man travels 150 miles in 4 days. How many miles does he travel each day?

20. Divide 90 by 720. Divide 49 by 8, and give the answer in decimal form.

GENERAL REVIEW

100. 1. How many pounds of butter at 35¢ a pound are worth as much as 1600 bushels of oats at 75¢ a bushel?

2. Paid $17,000 for a farm at the rate of $36 per acre. How many acres did I buy?

3. If 1 acre will produce 45 bu. of corn, how much will $\frac{2}{3}$ acres produce?

4. Find the cost of 144 yards of curbing at \$$6\frac{5}{8}$ per yard.

5. A man sold $\frac{3}{8}$ of his farm and had 100 acres left. How many acres had he at first? (Use sketch to help you.)

6. Find the cost of $\frac{3}{4}$ ton of hay at \$16.80 a ton.

7. If 3 yards of cloth can be bought for 50¢, what should be paid for 9 yards?

8. How many coats can be made from 32.4 yards of cloth, if it takes 2.7 yards for each coat?

9. Find the cost of .625 tons of coal at \$6.50 per ton.

10. If 36 boxes of raisins, each containing 36 pounds, cost \$194.40, what is the cost per pound?

11. What is the cost of 2 bushels of peas at 30¢ a peck?

12. The multiplier is 739 and the product is 671,012. What is the multiplicand?

13. In what time, at 75¢ a day, will a boy earn \$3.75?

14. How many pecks in $84\frac{1}{2}$ bushels?

15. If 63 lb. of tea cost \$34.65, how much will 15 lb. cost?

16. A man paid $16 for a coat, which was $\frac{2}{7}$ of all the money he had. How much money had he at first? (Sketch to help.)

17. 9 is one factor of 153. What is the other factor?

18. If a peck of peas cost 35¢, how much will 2 bushels cost at the same rate?

19. How many pounds of rice, at 8¢ a pound, can be bought for $1?

20. At $1.08 a gallon, what will 3 qt. of syrup cost?

21. Find the gain on 10 gallons of milk, bought at 15¢ a gallon and sold at 6¢ a quart.

22. Find the cost of 4 gal. milk at 4¢ a pint.

23. Sold 5 bu. of oats at 10¢ a peck. How much did I receive?

24. What will $2\frac{1}{2}$ gal. of oil cost at 6¢ a quart?

25. How many melons, at 25¢ each, can be bought for $3.50?

101. What we have learned about Fractions and Decimals.

1. A fraction is one or more of the equal parts of a unit. $\frac{2}{3}$ show 2 of the 3 equal parts of a unit.

2. In a fraction the number written below the line is called the denominator. It shows into how many parts the unit has been divided. In $\frac{3}{4}$, 4 is the denominator. It shows that the unit has been divided into 4 parts.

3. In a fraction, the number written above the line is called the numerator. It shows how many of the equal parts are used. $\frac{7}{8}$ show that 7 of the 8 equal parts have been used.

4. When a fraction and a whole number are written together, the number thus formed is called a mixed number. $2\frac{1}{2}$ is a mixed number. It means 2 whole units and $\frac{1}{2}$ of another unit.

5. To find a fractional part of a number, we divide it into the number of parts shown by the denominator, and take the number of such parts which the numerator requires.

In finding $\frac{2}{3}$ of 12, we divide 12 into 3 equal parts and take 2 of those equal parts.

6. To find the value of the whole unit when the value of one or more of its fractional parts is given, we find the value of each part, and multiply this by the number of parts in the whole unit. If the value of $\frac{2}{3}$ is given as $6, we find the value of $\frac{1}{3}$, which is $2, and then, as we know that it will take $\frac{3}{3}$ to make a whole unit, we multiply the $2 by 3 to get the value of the whole unit.

7. A decimal is some tenth part of a unit.

8. Decimals are written at the right of units' place, the value of each figure in the decimal de-

pending upon the number of places it is from units' place.

9. We add and subtract decimals as if they were whole numbers, and place the decimal point in the result under the points in the numbers used.

10. We multiply decimals as if they were whole numbers, and point off as many decimal places in the product as there are decimal places in both factors.

11. We divide decimals (describe the method which you have learned to use).

12. We can use a mixed number as one of the factors in multiplication, if we multiply by the whole and the fractional part of the mixed number separately and then add the products.

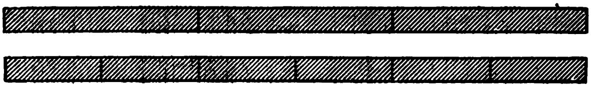
13. We find it helpful in working many hard examples to make a sketch which will help us to see what is given and what we are required to find.

14. When there are large or strange numbers used in an example, we read the example, using small and easy numbers in place of the large ones, not changing the words of the example. This makes it easier for us to tell what to do. After we know what to do with the numbers, we use the large ones given in the example.

FRACTIONS

102. 1. Which is the larger amount, $\frac{1}{2}$ lb. or $\frac{2}{4}$ lb.? (Sketch.)

Thirds and Sixths



2. Which is the larger, $\frac{1}{3}$ or $\frac{2}{6}$? (Sketch.)
3. Which is the larger, $\frac{1}{2}$ or $\frac{3}{6}$? (Sketch.)
4. Which is the larger, $\frac{2}{3}$ or $\frac{4}{6}$? (Sketch.)

If you made sketches as required above, or folded paper and compared the parts, you found that $\frac{1}{2}=\frac{2}{4}$, $\frac{1}{3}=\frac{2}{6}$, $\frac{1}{2}=\frac{3}{6}$, and $\frac{2}{3}=\frac{4}{6}$.

Study the examples given and see if this statement is true:

Both numerator and denominator may be multiplied by the same number without changing the value of the fraction.

Dividing both numerator and denominator of $\frac{4}{8}$ by 4, we have $\frac{1}{2}$. Did we change the value of the fraction?

Dividing both numerator and denominator of $\frac{3}{6}$ by 3, we have $\frac{1}{2}$. Dividing both numerator and denominator of $\frac{3}{9}$ by 3, we have $\frac{1}{3}$. Does it change the value of a fraction to divide both numerator and denominator by the same number?

Changing fraction to higher or lower terms.

Note to Teacher. It is better not to use the common term, "reduction," as reduction suggests change of value. To reduce anything, such as a debt, we make it less.

103. 1. Change $\frac{2}{3}$ to a fraction of the same value, having a denominator of 12. (Multiply both numerator and denominator by 4.) How can you tell whether to multiply by 4 or 5?

2. Change $\frac{6}{7}$ to 14ths. What number shall we use to multiply the numerator and denominator?

3. Change $\frac{4}{5}$ to 25ths.

4. Change $\frac{3}{8}$ to 24ths.

5. Change $\frac{3}{7}$ to 28ths.

6. Change $\frac{16}{24}$ to smaller terms. (The numerator and the denominator are called terms of a fraction.)

7. Change $\frac{18}{24}$ to smaller terms.

8. Change $\frac{49}{56}$ to smaller terms.

9. Change $\frac{128}{512}$ to smaller terms.

In such cases, where the numbers are large, we make use of several small divisors, dividing by 2, 2, 2, 2, etc., until the terms get small enough for us to see what single divisor we can use to complete the change.

10. Change $\frac{216}{504}$ to smaller terms.

ADDITION OF FRACTIONS

In adding fractions, if the denominators are alike, we add the numerators. Adding two thirds and one third is like adding $2 and $1. $\frac{2}{3}$ and $\frac{1}{3}$ make $\frac{3}{3}$, just as $2 and $1 make $3. If the denominators are not alike, we must change the terms of the fractions until they are alike before we can add. To add $\frac{1}{2}$ and $\frac{1}{3}$ we change the terms, so that $\frac{1}{2}$ becomes $\frac{3}{6}$ and $\frac{1}{3}$ becomes $\frac{2}{6}$. Then we add $\frac{3}{6}$ and $\frac{2}{6}$ and have $\frac{5}{6}$ *Ans.*

104. 1. Add $\frac{2}{3}$ and $\frac{3}{4}$.

$$\begin{array}{l} \frac{2}{3} = \frac{8}{12} \\ \frac{3}{4} = \underline{\frac{9}{12}} \\ \qquad \frac{17}{12} = 1\frac{5}{12} \text{ Ans.} \end{array}$$

2. Add $\frac{3}{4}$ and $\frac{4}{5}$.

$$\begin{array}{l} \frac{3}{4} = \frac{15}{20} \\ \frac{4}{5} = \underline{\frac{16}{20}} \\ \qquad \frac{31}{20} = 1\frac{11}{20} \text{ Ans.} \end{array}$$

3. Find the sum of $\frac{3}{5}$ and $\frac{3}{4}$.

4. Find the sum of $\frac{2}{3}$ and $\frac{3}{5}$.

5. Find the sum of $\frac{5}{6}$ and $\frac{3}{4}$.

$$\begin{array}{l|l} 16\frac{1}{2} & \frac{3}{6} \\ 24\frac{1}{3} & \frac{2}{6} \\ \hline 40\frac{5}{6} & \frac{5}{6} \end{array}$$

6. Find the sum of $16\frac{1}{2}$ and $24\frac{1}{3}$.

We add the whole numbers and the fractions separately. A convenient form of arrangement is shown.

7. Find the sum of $67\frac{2}{3}$ lb. and $24\frac{1}{2}$ lb.

8. What is the sum of $67\frac{1}{4}$ bu. and $45\frac{1}{2}$ bu.?

9. Add $24\frac{3}{4}$; $16\frac{1}{2}$; $42\frac{1}{4}$; 15; $25\frac{3}{4}$.

$$\begin{array}{r|l}
24\frac{3}{4} & \frac{3}{4} \\
16\frac{1}{2} & \frac{2}{4} \\
42\frac{1}{4} & \frac{1}{4} \\
15 & \\
25\frac{3}{4} & \frac{3}{4} \\
\hline
\textit{Ans.}\quad 124\frac{1}{4} & \frac{9}{4} = 2\frac{1}{4}.
\end{array}$$

10. Find the sum of $67\frac{2}{3}$; $45\frac{3}{4}$; $150\frac{1}{2}$; $62\frac{1}{4}$.

11. Find the sum of $678\frac{2}{3}$; $453\frac{1}{2}$; 457; $560\frac{1}{4}$; 870.

12. A grocer sold $47\frac{3}{4}$ lb. of sugar to one man, $83\frac{3}{5}$ lb. to another, and $68\frac{1}{2}$ lb. to another. How many pounds did he sell in all?

13. A man bought 3 pieces of cloth: one contained $45\frac{3}{4}$ yd., another $63\frac{5}{8}$ yd., and the other $56\frac{1}{2}$ yd. How many yards did he buy?

14. If you travel $85\frac{7}{8}$ mi. in one day, $95\frac{3}{4}$ mi. the second, and $115\frac{1}{2}$ mi. the third, how far will you travel in the three days?

15. A farmer had $246\frac{7}{10}$ A. of land, and bought $57\frac{1}{4}$ A. How many did he then have?

16. There are $16\frac{3}{4}$ bu. of oats in one bin, $45\frac{1}{8}$ bu. in another, and $120\frac{1}{2}$ bu. in the third. How many bushels in all of the bins?

17. Find the sum of $62\frac{1}{4}$; $85\frac{1}{2}$; $7\frac{5}{8}$; $645\frac{3}{4}$.

18. Find the sum of $670\frac{1}{4}$; $75\frac{2}{3}$; $85\frac{1}{2}$; $15\frac{1}{3}$.

SUBTRACTION OF FRACTIONS

In subtraction of fractions we have to change the terms if they are not alike, so that the denominators become alike, and then subtract the numerators.

105. 1. Subtract $\frac{1}{4}$ from $\frac{2}{3}$.

$$\begin{array}{rcl} \frac{2}{3} & = & \frac{8}{12} \\ -\frac{1}{4} & = & \underline{\frac{3}{12}} \\ & & \frac{5}{12} \quad \textit{Ans.} \end{array}$$

2. Subtract $\frac{3}{4}$ from $\frac{7}{8}$.

3. From $\frac{7}{9}$ take $\frac{2}{3}$.

4. Subtract $\frac{5}{8}$ from $\frac{3}{4}$.

5. Subtract $16\frac{1}{2}$ from $24\frac{3}{4}$.

$$\begin{array}{rr|l} & 24\frac{3}{4} & \frac{3}{4} \\ - & \underline{16\frac{1}{2}} & \underline{\frac{2}{4}} \\ \textit{Ans.} & 8\frac{1}{4} & \frac{1}{4} \end{array}$$

This is easy when the fraction in the upper number is larger than the other fraction. When it is smaller, we have to work very carefully. See if you can understand the work shown for the next example.

6. Subtract $8\frac{3}{4}$ from $12\frac{1}{2}$.

$$\begin{array}{rr|l} & 12\frac{1}{2} & \frac{2}{4} \\ - & \underline{8\frac{3}{4}} & \underline{\frac{3}{4}} \\ \textit{Ans.} & 3\frac{3}{4} & \frac{3}{4} \end{array}$$

Look at this work also.

$$\begin{array}{rr} & 122 \\ - & \underline{83} \\ & 39 \end{array}$$

What did we do when we could not subtract the 3 from the 2 when we were subtracting whole numbers? What did we do when we could not subtract $\frac{3}{4}$ from $\frac{2}{4}$ when we were subtracting the fractions? Some children find the subtraction of fractions so hard that they have to cross out the figures when they "borrow" and write in the new ones like this:

$$\begin{array}{r|l} 11 & \frac{6}{4} \\ \cancel{1}\cancel{2}\frac{1}{2} & \frac{2}{4} \\ -\ 8\frac{3}{4} & \frac{3}{4} \\ \hline \textit{Ans.}\ \ 3\frac{3}{4} & \frac{3}{4} \end{array}$$

7. Subtract $8\frac{7}{8}$ from $14\frac{1}{2}$.

8. Subtract $24\frac{5}{6}$ from $48\frac{1}{3}$.

9. Subtract $15\frac{2}{3}$ from $45\frac{2}{9}$.

10. Subtract $25\frac{7}{8}$ from $84\frac{3}{4}$.

11. A farmer had 150 A. of land, and sold $45\frac{3}{4}$ A. How many acres had he left?

12. A bin has 650 bu. in it, and $54\frac{1}{2}$ bu. are taken out. How many bushels remain in the bin?

13. There were $67\frac{1}{2}$ T. of hay in the barn, and $45\frac{3}{4}$ T. are sold. How many tons remained in the barn?

14. A man bought $5\frac{1}{2}$ lb. of sugar and then sold
[illegible] How many pounds had he left?

15. [illegible] l. of oil in a barrel, and
[illegible] How many gallons re-

16. From a cheese weighing 54 lb., $5\frac{3}{16}$ lb. are cut. How many pounds of cheese remain?

17. There were 45 bu. of oats in a bin, and the farmer used $12\frac{3}{4}$ bu. for seed. How many bushels are there left in the bin?

MULTIPLICATION OF FRACTIONS

We have learned to use one factor in fractional form in multiplication; we are now to learn to use both factors in fractional form.

106. 1. Find the cost of $\frac{2}{3}$ yd. of cloth at $\$\frac{3}{8}$ per yard.

Long way

1 yd. costs $\$\frac{3}{8}$
$\frac{1}{3}$ yd. costs $\$\frac{1}{8}$
$\frac{2}{3}$ yd. costs $\$\frac{2}{8}=\$\frac{1}{4}$ *Ans.*

Short way

$$\frac{\overset{1}{\cancel{3}}}{8}\times\frac{2}{\underset{1}{\cancel{3}}}=\frac{2}{8}=\frac{1}{4}.$$

$\$\frac{1}{4}$ *Ans.*

2. Find the cost of $\frac{2}{3}$ lb. of tea at $\$\frac{3}{4}$ a pound.

Long way

1 lb. costs $\$\frac{3}{4}$
$\frac{1}{3}$ lb. costs $\$\frac{1}{4}$
$\frac{2}{3}$ lb. costs $\$\frac{2}{4}=\$\frac{1}{2}$ *Ans.*

Short way

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}=\frac{1}{2}.\quad \$\frac{1}{2}\ \textit{Ans.}$$

3. Find the cost of $\frac{2}{5}$ bu. at $\$\frac{5}{8}$ a bushel.

Short way

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}}=\frac{1}{4}.\quad \$\frac{1}{4}\ \textit{Ans.}$$

4. Multiply $\frac{8}{9}$ by $\frac{3}{4}$.

Short way

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} = \frac{2}{3} \quad \textit{Ans.}$$

5. Find the cost of $\frac{3}{8}$ lb. of cheese at 16¢ a pound.

Short way

$$\frac{\overset{2}{\cancel{16}}}{1} \times \frac{3}{\underset{1}{\cancel{8}}} = 6. \quad 6\text{¢} \ \textit{Ans.}$$

From the study of the above examples, we see that we can multiply by a fraction by dividing the multiplicand into the number of parts indicated by the denominator of the multiplier, and then taking the number of such parts which the numerator requires.

We can do this in a short way by multiplying the corresponding terms of the fractions together. We can shorten the operation by rejecting equal factors from both numerator and denominator. This rejecting of equal factors is called *Cancellation.*

6. At $\$\frac{9}{10}$ a gallon, what will $\frac{3}{8}$ gallons cost?

7. A man owning $\frac{7}{8}$ of a ship sold $\frac{2}{3}$ of his share. What part of the ship did he sell?

8. If a horse travels $6\frac{3}{4}$ miles in an hour, how far will he travel in $\frac{4}{5}$ hours?

Note. 6 = 24 fourths. $6\frac{3}{4} = \frac{27}{4}$. $\frac{27}{4} \times \frac{4}{5}$.

9. What will $\frac{3}{8}$ of a bushel of pears cost at \$ $\frac{3}{4}$ a bushel?

10. What will $2\frac{1}{4}$ bushels of peaches cost at \$$2\frac{4}{5}$ a bushel? ($\frac{9}{4} \times \frac{14}{5}$.)

11. If a family burns $1\frac{3}{8}$ tons of coal each month, how many tons will it burn in $2\frac{3}{4}$ months?

12. Multiply $\frac{16}{27}$ by $\frac{18}{24}$.

13. Bought $\frac{1}{2}$ bu. of beans and planted $\frac{3}{4}$ of them. What part of a bushel did I plant?

14. A man having $\frac{1}{2}$ of an acre of land sold $\frac{3}{4}$ of it. What part of an acre did he sell?

15. At $6\frac{2}{3}$ dollars a ton, what must I pay for $8\frac{3}{4}$ tons of coal?

DIVISION OF FRACTIONS

107. $1 \div \frac{1}{2} = 2$ or $\frac{2}{1}$. $1 \div \frac{1}{3} = 3$ or $\frac{3}{1}$. $1 \div \frac{1}{4} = 4$ or $\frac{4}{1}$.

1. Divide $\frac{3}{4}$ by $\frac{1}{2}$.

If 1 ÷ by $\frac{1}{2} = \frac{2}{1}$, $\frac{3}{4}$ of 1 ÷ by $\frac{1}{2}$ will equal $\frac{3}{4}$ of $\frac{2}{1}$. $\frac{3}{4} \times \frac{2}{1} = \frac{6}{4}$ or $1\frac{1}{2}$ *Ans.*

2. Divide $\frac{7}{8}$ by $\frac{2}{3}$.

$1 \div \frac{2}{3} = \frac{3}{2}$. $\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16}$ or $1\frac{5}{16}$ *Ans.*

3. Divide $\frac{3}{8}$ by $\frac{5}{6}$. $1 \div \frac{5}{6} = \frac{6}{5}$. $\frac{3}{8} \div \frac{5}{6} = \frac{3}{8} \times \frac{6}{5} = ?$

4. Divide $\frac{6}{7}$ by $\frac{4}{9}$. $\frac{6}{7} \times \frac{9}{4} = ?$

Short way to divide by a fraction: *Invert the divisor and use it as a multiplier.*

5. How many thirds are there in 4 whole things?

Short way	Long way
$\frac{4}{1}\times\frac{3}{1}=12$ *Ans.*	There are 3 thirds in 1. In 4 times 1 there are 4 × 3 thirds or 12 thirds. 12 *Ans.*

6. A man earns $2\frac{3}{4}$ a day. How long will it take him to earn $33?

$2\frac{3}{4}=\frac{11}{4}$. $\frac{33}{1}\div\frac{11}{4}=\frac{33}{1}\times\frac{4}{11}$. $\frac{33}{1}\div\frac{4}{11}=?$ *Ans.*

7. If a man earns $2.75 a day, how long will it take him to earn $33?

8. How many barrels of apples, each holding $2\frac{3}{8}$ bushels, can be filled with 18 bushels of apples?

9. How many pounds of beef, at $11\frac{3}{4}$ ¢ a pound, can be bought for $87\frac{1}{2}$ ¢?

10. If 4 lb. cost $\frac{3}{5}$ dollars, what will 1 lb. cost?

$$\frac{3}{5}\div 4=\frac{3}{5}\times\frac{1}{4}.$$

11. A man divided $\frac{3}{4}$ acres into 3 equal lots. What part of an acre was there in each lot?

12. If $\frac{2}{3}$ lb. of tea be equally divided among 3 persons, what part of a pound will each receive?

13. $\frac{1}{2}$ of a ship is owned by five men, each owning an equal part. What part of the ship did each man own?

14. How many times is $\frac{1}{7}$ contained in 12?

15. How many times is $\frac{2}{3}$ contained in 18?

16. How many aprons can be made from $4\frac{1}{2}$ yds. of muslin if each apron requires $1\frac{1}{2}$ yds.?

17. Divide $18\frac{2}{3}$ by $\frac{3}{4}$.

RELATION OF WHOLE NUMBERS, FRACTIONS, AND DECIMALS

A whole number, often called an integer, shows the number of whole things used. 1, 2, 3, 4, 45, 48, etc., are whole numbers.

A fraction shows some part of an integer. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, etc., are fractions, and show that some whole number has been divided into parts, and a certain number of those parts are shown by the numerator of the fraction.

A decimal is some tenth part of a number, and is like a fraction in that it is part of a whole thing. It can be written in fractional form by writing for its denominator the number which is shown by its name as a decimal.

.5 is read 5 tenths, and can be written $\frac{5}{10}$; .04 is read 4 hundredths, and can be written $\frac{4}{100}$; .006 is read 6 thousandths, and can be written $\frac{6}{1000}$.

In changing a decimal to fractional form, we write 1 and as many ciphers as there are decimal places in the decimal given for its denominator.

108. Change the following decimals to fractional form:

1. .04 $\frac{4}{100}$ *Ans.* .004 $\frac{4}{1000}$ *Ans.* .44 $\frac{44}{100}$ *Ans.*

2. .453, .032, .345, .54, .089, .765, .452.

3. 1.04 $1\frac{4}{100}$ *Ans.* 3.005 $3\frac{5}{1000}$ *Ans.*

(The number at the left of the decimal point is an integer, and must be written as an integer when in fractional form.)

109. Change the following fractions to integers or decimals:

1. $\frac{4}{2}$ 2 *Ans.* $\frac{12}{3}$ 4 *Ans.* $\frac{16}{4}$ 4 *Ans.*

2. $\frac{5}{2}$ 2.5 *Ans.* We divide, carrying out the remainder in division and have .5. $\frac{5}{2}$ in fractional form is written as a mixed number, $2\frac{1}{2}$, or as a decimal, 2.5.

$$\begin{array}{l} .875 \textit{ Ans.} \\ 8\overline{)7.000} \end{array}$$

3. $\frac{7}{8}$ $8\overline{)7.000}$

4. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

(These fractions are so often used in their decimal form that you should remember both forms, and be able to change from one to the other at sight.)

5. Find the cost of $\frac{1}{8}$ lb. at \$.16 a pound. Using $\frac{1}{8}$ as a fraction, $\frac{16}{1}\times\frac{1}{8}=2$¢ *Ans.* Using the value of $\frac{1}{8}$ as a decimal,

$\frac{1}{8}=.125$,

$$\begin{array}{r} .125 \\ 16 \\ \hline 750 \\ 125 \\ \hline 2.000 \end{array}$$

2¢ *Ans.*

We learn that fractions and decimals are much alike, and that we can change from one form to the other very easily.

In working problems, we should use either the fractional or the decimal form, as is most convenient.

6. Find the value of $\frac{7}{8}$ lb. of tea at 35¢ a pound.

7. Find the cost of $\frac{3}{4}$ yd. of cloth at 50¢ a yard.

SUMMARY OF FRACTIONS

110. 1. A fraction is one or more of the equal parts of anything.

2. The denominator shows into how many equal parts the thing has been divided.

3. The numerator shows how many of the equal parts are taken.

4. The terms of a fraction are the numerator and the denominator.

5. A mixed number is a whole number and a fraction taken together.

6. A mixed number can be changed to fractional form by finding the number of parts in the whole number and adding them to the number of parts shown in the fraction.

7. An improper fraction is one whose numerator is larger than its denominator. It can be changed to a mixed number by dividing the numerator by the denominator.

8. The terms of a fraction may be multiplied or divided by any number without changing the value of the fraction.

9. We call fractions which have like denominators similar fractions.

10. Before we can either add or subtract fractions we must change their form so that their denominators will be alike.

11. To add fractions, we change them to similar fractions and then add their numerators.

12. To subtract fractions, we change them to similar fractions and subtract one numerator from the other.

13. In multiplication of fractions, we use both factors in fractional form, and then multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.

14. We may reject equal factors from both numerator and denominator of a fraction when changing its form to lower terms. This is often called "reduction to lowest terms."

15. We can reject equal factors from the denominators and the numerators, when multiplying fractions, to shorten the operation. This is called cancellation.

16. To divide fractions, we invert the divisor and use it as a multiplier.

17. We may write a whole number in fractional form by using it as the numerator of a fraction whose denominator is 1.

18. To find a fractional part of a number, we multiply the number by the fraction.

$$\frac{2}{3} \text{ of } 12 = \frac{2}{3} \times \frac{12}{1}.$$

19. To find the value of the whole number when the value of a part is given, we divide the value of the fractional part by the fraction. $\frac{1}{4}$ lb. costs 12¢, what will 1 lb. cost? $\frac{12}{1} \div \frac{1}{4} = 48$. 48¢ *Ans.*

20. We can change a fraction to decimal form by dividing its numerator by its denominator, carrying out for the decimal places. $\frac{1}{4} = ?$ decimal.

$$\begin{array}{rl} .25 & \textit{Ans.} \\ 4\overline{)1.00} & \end{array}$$

PROBLEMS IN REVIEW OF FRACTIONS AND DECIMALS

Suggestions to the pupils. 1. Make an abstract of the problem, writing in brief what is given and what is required, writing similar facts under each other.

2. See what terms the answer will express, and indicate it after the question mark, thus ? lb. *Ans.*, ? bushels, *Ans.*, ? $ *Ans.*, etc. Then do the work in abstract, and write the number thus found for the question mark just used.

3. Make sketches and use folded paper to help you understand what is given and what you are asked to find.

4. When the numbers are large or strange, substitute small and familiar terms for them, and read the same words with these easy numbers, and find what you are to do, then use the figures in the book just as you would use the easy figures.

REVIEW PROBLEMS IN FRACTIONS AND DECIMALS

111. 1. $\frac{1}{5}$ of the expenses of an excursion were paid by Mr. Smith, $\frac{1}{8}$ of them by Mr. Brown, and the rest, which was $\$27$, by the rest of the party. What were the expenses of the excursion?

$$\begin{array}{l} \frac{1}{5}\text{ of ex.} \\ \frac{1}{8}\text{ of ex.} \\ +\,?\text{ of ex. which is } \$27 \\ \hline \text{All of ex. which is } \$? \end{array} \qquad \$\text{——}\ \textit{Ans.}$$

$$\begin{array}{r} \frac{1}{5} = \frac{8}{40} \\ +\frac{1}{8} = \frac{5}{40} \\ \hline \frac{13}{40}\text{ part paid.} \end{array} \qquad \begin{array}{rl} \frac{40}{40} & = \text{all of expense.} \\ -\frac{13}{40} & = \text{part paid.} \\ \hline \frac{27}{40} & = \$27\text{ to be paid.} \end{array}$$

If $\frac{27}{40} = \$27$, $\frac{40}{40} = \$27 \div \frac{27}{40} = \$40$ *Ans.*

2. A man spent $\frac{1}{3}$ of his life traveling in America, $\frac{1}{4}$ of it in Europe, and the rest, which was 25 years, at home. How old was the man? (Arrange work in orderly way, as shown for the first example.)

3. How much will 5 masons earn in $4\frac{1}{2}$ days, at $\$3\frac{3}{4}$ each per day?

4. After spending $\frac{3}{4}$ of my money, I had $\$35$ left. How much had I at first? (Folded paper may help you.)

5. A man spent $\frac{1}{5}$ of his money and $\$25$ more, and then had $\$225$ left. How much money had he at first?

6. If a man walks $8\frac{1}{2}$ miles in $2\frac{2}{5}$ hours, how far can he walk in $5\frac{5}{8}$ hours? (Try suggestion No. 4.)

7. If $\frac{5}{6}$ lb. of butter cost $.25, what is the price per pound?

8. If $\frac{3}{5}$ lb. of cheese cost 9¢, what will 4 lb. cost?

9. If $5\frac{1}{4}$ lb. of nuts are divided equally among 7 boys, what part of a pound will each receive?

10. A farmer sold $\frac{1}{5}$ of his corn at one time, $\frac{2}{3}$ at another, and then had 80 bushels left. How much corn had he at first?

11. If $\frac{3}{8}$ of a farm is valued at $$1729\frac{1}{2}$, what is the value of the whole farm?

12. If $\frac{2}{3}$ of a farm is worth $9000, what is $\frac{5}{12}$ of it worth?

13. A coal dealer sold $\frac{2}{7}$ of his coal for $90, at $6 per ton. How many tons of coal had he?

14. A man having $24 gave $\frac{2}{3}$ of his money for clover seed at $$5\frac{1}{3}$ a bushel. How many bushels did he buy?

15. If $$7\frac{1}{2}$ will buy $3\frac{1}{2}$ cords of wood, how many cords can be bought for $$31\frac{1}{2}$?

16. If 64 tons of iron cost $4816, how many tons can be bought for $7130?

17. If an acre of land produces 127.25 bushels of potatoes. how many bushels will $4\frac{3}{8}$ acres produce?

18. If $\frac{3}{8}$ lb. of cheese cost 6¢, what will .75 lb. cost?

19. If .125 lb. of butter cost 4¢, what will $3\frac{1}{2}$ lb. cost?

20. If $\frac{7}{8}$ of a farm is worth \$7500, what is .875 of it worth?

SHORT METHODS

112. Use of business fractions.

The following fractions are so often used in business that they are often called business fractions.

$$\frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{4},\ \frac{3}{4},\ \frac{1}{5},\ \frac{2}{5},\ \frac{3}{5},\ \frac{4}{5},\ \frac{1}{6},\ \frac{5}{6},\ \frac{1}{8},\ \frac{3}{8},\ \frac{5}{8},\ \frac{7}{8}.$$

The following table shows the value of each as parts of a dollar:

$\$\frac{1}{2} = 50¢$
$\$\frac{1}{3} = 33\frac{1}{3}¢$
$\$\frac{2}{3} = 66\frac{2}{3}¢$
$\$\frac{1}{4} = 25¢$
$\$\frac{3}{4} = 75¢$
$\$\frac{1}{5} = 20¢$
$\$\frac{2}{5} = 40¢$
$\$\frac{3}{5} = 60¢$
$\$\frac{4}{5} = 80¢$
$\$\frac{1}{6} = 16\frac{2}{3}¢$
$\$\frac{5}{6} = 83\frac{1}{3}¢$
$\$\frac{1}{8} = 12\frac{1}{2}¢$
$\$\frac{3}{8} = 37\frac{1}{2}¢$
$\$\frac{5}{8} = 62\frac{1}{2}¢$
$\$\frac{7}{8} = 87\frac{1}{2}¢$

1. Find the cost of 48 yards at $12\frac{1}{2}$¢ a yard.

$12\frac{1}{2}¢ = \$\frac{1}{8}$. 48 yd. at \$1 costs \$48.
48 yd. at $\$\frac{1}{8}$ costs \$6. *Ans.*

2. Find the cost of 96 lb. of cheese at $16\frac{2}{3}$¢ a pound.

3. Find the cost of 80 gal. molasses at $37\frac{1}{2}$¢ a gallon.

4. Find the cost of 18 pairs of gloves at 66⅔¢ a pair.

5. Cost of 12 ties at 75¢ each.

6. Cost of 36 —— at 83⅓¢.
   Cost of 24 —— at 37½¢.

7. Cost of 15 —— at 33⅓¢.
   Cost of 16 —— at 25¢.

8. Cost of 24 —— at 75¢.
   Cost of 8 —— at 87½¢.

9. Cost of 8 —— at 75¢.
   Cost of 24 —— at 16⅔¢.

10. Cost of 9 —— at 66⅔¢.
    Cost of 16 —— at 62½¢.

113. Sometimes we can use a short method when the quantity bought is an easy fractional part of 100.

1. Find the cost of 12½ lb. at 16¢ a pound.

100 lb. at 16¢ cost $16.

⅛ hundred lb. at 16¢ costs $2 *Ans.*

2. Find the cost of 75 bushels of oats at 36¢ a bushel.

100 bu. at 36¢ costs $36.

75 or ¾ hundred bu. at 36¢ cost $27 *Ans.*

3. Find the cost of 150 lb. of cheese at 16¢ a pound.

100 lb. at 16¢ cost $16.

1½ hundred lb. cost 1½ of $16 or $24 *Ans.*

4. Find the cost of 225 lb. of butter at 24¢ ($2\frac{1}{4}$ hundred lb.).

5. Find the cost of 250 lb. at 18¢.

6. Find the cost of 125 lb. at 16¢.

7. Find the cost of 75 lb. at 12¢.

8. Find the cost of $87\frac{1}{2}$ lb. at 16¢.

9. Find the cost of $33\frac{1}{3}$ lb. at 48¢.

10. Find the cost of $16\frac{2}{3}$ lb. at 18¢.

Caution. Do not try to use short methods unless you are sure you will make no mistakes in using them. It is better to take a little longer and get the right answer than to try to use a short way and make a mistake.

DRILL EXERCISES IN REVIEW

114. Find the cost of:

1. 695 lb. @ 20¢.
2. 93 yd. @ $33\frac{1}{3}$¢.
3. $12\frac{1}{2}$ yd. @ 48¢.
4. 25 bu. @ 32¢.
5. 150 lb. @ 24¢.
6. 16 lb. @ $37\frac{1}{2}$¢.
7. 48 bu. @ $1.50.
8. 75 bu. @ 48¢.
9. $12\frac{1}{2}$ yd. @ 64¢.
10. 250 yd. @ 32¢.
11. $\frac{3}{8}$ yd. @ 24¢.
12. 35 lb. @ 40¢.
13. 56 bu. @ $37\frac{1}{2}$¢.
14. 24 lb. @ $16\frac{2}{3}$¢.
15. 150 doz. @ 18¢.
16. 48 lb. @ 75¢.
17. 12 lb. @ $1.75.
18. 36 lb. @ $.75.
19. 250 bu. @ $4.50.
20. 25 bu. @ 72¢.
21. 75 bu. @ 84¢.
22. 96 bu. @ $12\frac{1}{2}$¢.
23. $12\frac{1}{2}$ bu. @ 96¢.
24. 75 qt. @ 16¢.

115. A short method, when one of the factors in multiplication is nearly the same as some factor which can be used more easily in computation.

1. Find the cost of 7 yd. of cloth at 98¢ a yard. (At $1 a yard, $7; at 2¢ less than $1 a yard, 14¢ less than $7. $6.86 *Ans.*)

2. Find the cost of 97 bushels of oats at 32¢ a bushel. (100 bu. at 32¢, $32. 3 bu. less, 96¢ less, $31.04 *Ans.*)

3. Find the cost of 48 bu. at 26¢ a bushel. (48 bu. @ $$\tfrac{1}{4}$ = $12. 1¢ a bu. more, 48¢ more. $12.48 *Ans.*)

4. Find the cost of 18 lb. at 52¢ a pound. (At 50¢, $9. 2¢ lb. more, 36¢ more. $9.36 *Ans.*)

5. Find the cost of 14 yards at 98¢ per yard.
6. Find the cost of 97 bushels at 32¢ per bushel.
7. Find the cost of 14 yards at $ 2.98 per yard.
8. Find the cost of 298 lb. at 45¢ per lb.
9. Find the cost of 3½ yards at 98¢ a yard.

Except in so-called bargain stores, where goods are often marked down from $1.00 to 98¢, etc., this method has little value, as the exercises will not often come in convenient form for its use.

Time is not usually as important as the correct result, therefore it is not wise to depend upon the use of short methods except for drill, where little is at stake should an error be made.

116. BILLS

Brooklyn, N.Y., June 10, 1909.

Mr. John Smith,
Mt. Vernon, N.Y.,
Bought of Frederick Loeser & Co.,
General Merchants.

4 lb. Coffee,	@ $.35	$1	40
½ lb. Tea,	@ .60		30
25 lb. Sugar,	@ .06	1	50
8 lb. Raisins,	@ .12½	1	00
1 sack Flour,		1	25
1 barrel Apples,		3	50
Received payment,		$ 8	95

Frederick Loeser & Co.
per A. L. S.

The above is the usual form of a statement or bill of goods sold which Frederick Loeser & Co. would make and send to Mr. Smith. They would not write "Received payment" until Mr. Smith had paid them for the goods. Business men often use a stamp with the word *Paid* and a blank for the date and the name of the one receiving the money.

The letters "A. L. S.," used in the example above, show that one whose name is shown by those letters received the money for the firm.

1. Make out bills in proper form from the following memoranda:

Boston, Mass., Jan. 4, 1909. Mrs. Samuel Adams, who lives at No. 34 Broad St., bought of

James Smith, Grocer, 15 lb. butter @ 28 ¢; 10 doz. eggs @ 24 ¢; 4 lb. cheese @ 18 ¢; $\frac{1}{2}$ lb. coffee at 36 ¢.

When the groceries were delivered, Mrs. Adams paid the driver for them. How should he receipt for the payment? Who should keep the bill, the driver or Mrs. Adams?

2. Mrs. James Brown bought of Smith Brothers, Atlanta, Ga., on Dec. 23, 1908, 2 dressed turkeys, weighing 12$\frac{1}{2}$ and 13 lb. at 22 ¢ a pound; 3 lb. lamb chops at 20 ¢ a lb. She paid the clerk, John Adams, for them.

3. Make out a bill for the following, using names known to you. Bought on March 4, 1909, 8$\frac{1}{2}$ lb. lard at 12 ¢; 4 cans corned beef at 16 ¢; 14$\frac{1}{2}$ lb. ham at 16 ¢; 2 lb. sausage at 12$\frac{1}{2}$ ¢. The bill has not yet been paid.

4. Make out a bill from the following, supplying all facts not given here:

Bought 3$\frac{1}{2}$ yd. satin at $1.50; 2 yd. flannel at 50 ¢; 1$\frac{1}{2}$ yd. canvas at 20 ¢; 1$\frac{1}{2}$ yd. ribbon at 23 ¢. Paid.

5. 5 lb. coffee at 32 ¢; 2 lb. tea at 45 ¢; 8 lb. ham at 15 ¢; 7 lb. lard at 8 ¢; 3$\frac{1}{2}$ lb. butter at 32 ¢; 3 qt. currants at 7 ¢; 2 doz. eggs at 30 ¢; $\frac{3}{4}$ lb. cheese at 16 ¢.

The bill was paid, and the money received by the driver, James S. Brown.

117. BILL FOR WORK AND MATERIALS

Albany, N.Y., Jan. 12, 1909.

Mr. James Brown,

To Samuel Smith, Dr.

To 3 days, labor repairing house,	@ $3.50	$10	50
Lumber furnished,		2	00
Paint furnished,		1	00
Nails and screws furnished,			75
		$14	25

Paid Jan. 15, 1909.

Samuel Smith.

1. Write out in proper form the following statement:

Mr. John Smith, a plumber, repaired the plumbing in the bath room of Mr. Isaac Lewis, finishing the work on May 12, 1908. He worked 15 hours, at 40¢ an hour, and used a helper for the same time, charging 10¢ an hour for the helper. He furnished the following materials:

1 tank, 6 gallons, $6.50; 10 feet pipe at 25¢ a foot; solder for 50¢. The bill was not paid, but was sent by mail to the owner of the house.

2. Write out in proper form the following:

James Brown worked for Abraham Stern on his lawn and garden as follows: Mowing lawn, 75¢; planting seeds, 50¢; grading walks, 50¢. He finished work on June 10, 1909. He left the bill at the house, as he did not see Mr. Stern.

3. Write out the following in proper form:

Mr. Frank Jones worked for Mr. David Smith as follows: Repairing fence, 4 hours at 25¢ an hour; making new gate, 1 day at $2.00; materials furnished, $1.50.

The work was finished on March 15, 1909, and bill paid on that day.

4. Make out several bills from circumstances which you may know, or if you do not know the facts, write in some that might be true.

118. BILL FOR RENT

BROOKLYN, N.Y., April 1, 1909.

MR. ISAAC BLOTZ,

TO DAVID R. JONES, DR.

To 1 month's rent, March, 1909, of No. 1426 Fifty-third St.,	$ 45.00

Received payment,

DAVID R. JONES.

1. Make bill for rent for some house which you know.

If you were to mail the bill, would you receipt it when you mailed it to the man? When he sent you the money, should he send you the bill also? If he did, what would you do with it?

2. Arrange the following into proper bill form:

On May 12, 1910, Mr. Samuel Jones paid the rent of "Sunnyside Cottage" for the month of May, paying $75 to the owner of the cottage, Mr. James Adams, who lives in Albany, New York.

119. SIMPLE FORM FOR AN ACCOUNT

JAMES S. BROWN, DR. CR.

1909 March	8	To 3 lb. butter,	@ $.32		96		
		2 lb. cheese,	@ .15		30		
		½ lb. coffee,	@ .35		18		
		¾ lb. tea,	@ .50		38		
"	9	To 1 sack flour,		1	50		
"	12	To 1 barrel apples,		3	50		
"	13	By cash paid on account,				3	00
"	15	To ½ doz. bars soap,	@ .44		22		
		4 doz. eggs,	@ .40	1	60		
"	18	To 2 qt. berries,	@ .12		24		
		½ bu. potatoes,	@ .90		45		
"	20	By cash paid on account,				5	00
"	22	Paid balance in cash,				1	33
				9	33	9	33

Notes. The person is debit to what he receives, and credit by what he gives.

The difference of the total debits and credits will show what more should be paid.

When an article costs a fraction of a cent more than a certain number, the merchant usually charges the next number for it. If the cost is 13½ cents, some would charge 13¢, but most merchants would charge 14¢.

An account for work done is kept on the same form of ruling, and the dates and other matters entered in the same way.

The man who receives the benefit of the work is debited for it. If you were working for John

R. Doe, you would write his name at the head of your account, as follows:

JOHN R. DOE, DR. CR.

When the one who is credited desires the other to pay, he should send a statement or bill, as we have learned to do for other matters in which we wish to collect money.

Write out some accounts against persons whom you know.

Write out such an account as the grocer would have against you.

120. SIMPLE FORM FOR A CASH BOOK

		CASH	RECEIVED		PAID	
1909 March	1	Balance on hand,	50	00		
		Rec. for work,	2	50		
"	2	Paid for shoes,			3	50
"	3	Rec. for 3 pk. potatoes,		75		
		Paid for repairs to clothes,			1	25
"	5	Paid for an umbrella,			2	50
"	6	Rec. for work,	7	50		
"	8	Paid for overcoat,			15	00
"	9	Paid for ½ ton of coal,			3	50
"	10	Rec. for 25 bu. oats, @ 50¢	12	50		
"	13	Paid for rent of house, Feb., 1909,			15	00
March	15	Balance,			32	50
			73	25	73	25
March	15	Balance on hand,	32	50		

Notes. Cash is treated just as if it was a person, and what it receives is placed in the first or debit column. What it gives is placed to its credit.

We can find what cash we should have on hand at any time by footing the two columns and subtracting the credit footing from the debit footing. The debit or received column will always have the larger footing. Why?

1. Rule some paper, and write out the following in proper form for a cash book:

I began to keep a cash book, and counted my money and found that I had $ 5.25. I began to keep the cash book on April 5, 1909. On April 5 I bought a knife for 45¢ and sold a pair of skates for $ 1.50. On April 8 I worked for a man and earned $ 1.25, which he paid me. I bought some candy for 15¢, and paid car fare twice, costing me 10¢. On April 10 I bought a new pair of shoes for $ 2.50. Balance the book, and find how much money I should have Saturday night, April 10, 1909.

GENERAL REVIEW

121. 1. The distance between two places is 50 miles. How long will it take a bicyclist to ride the distance, if he averages $9\frac{1}{2}$ miles per hour?

2. If 75 barrels of apples cost $ 375, what will 50 barrels cost?

3. John spent 57¢, which was $\frac{3}{5}$ of his money. How much money did he have? How much did he spend? How much had he left?

4. If $\frac{3}{4}$ acre produces 125 bushels of potatoes, how many bushels will $1\frac{1}{2}$ acres produce?

5. A man owned $\frac{2}{5}$ of a factory and sold $\frac{1}{3}$ of his share for $ 350. What was the whole factory worth at that rate?

6. The product of two numbers is 3.35. One of the numbers is .25; what is the other number?

7. If $\frac{3}{4}$ barrel costs $ 4.20, what will 5 barrels cost?

8. If 16 bushels cost $ 12, what will 4 bushels cost?

9. At the rate of 7 lb. for 50¢, what will 21 lb. cost?

10. At the rate of "6 for a quarter," how many oranges can you buy for 75¢?

11. Change to fractional form, 6.25; .875; 4.125.

12. Change to decimal form, $\frac{3}{4}$; $1\frac{3}{8}$; $3\frac{4}{5}$; $16\frac{7}{8}$.

13. Bought 80 lb. of sugar at $6\frac{1}{4}$¢; $48\frac{1}{2}$ lb. tea at 36¢; 124 lb. coffee at 16¢. Find the total cost.

14. At $ $1\frac{1}{4}$ a yard, how many yards of carpet can be bought for $ 30?

15. What will 9 lemons cost at 40¢ a dozen?

16. The sum of two fractions is $\frac{17}{18}$. One fraction is $\frac{2}{3}$; what is the other fraction?

17. The product of two numbers is $\frac{5}{8}$. One number is $1\frac{1}{8}$; what is the other?

18. One coat requires $2\frac{3}{8}$ yd. of cloth. How many coats can be made from $35\frac{5}{8}$ yd.?

19. $\frac{3}{8}$ of a farm is valued at $ 2400. What is the value of the whole farm at that rate?

20. For 5¢ I can buy 3 apples. How many should I buy for 35¢?

21. For 5¢ I can duy 3 apples. How much should I pay for 12 apples?

22. Find the cost of $\frac{3}{4}$ yd. of muslin at $12\frac{1}{2}$¢ a yard.

23. A house was sold for $ 1800, which was $\frac{3}{5}$ of its cost. How much was lost?

24. At 16¢ a pound, how many pounds of steak can be bought for 84¢?

25. A man earns $ 21 in a week (6 days). At that rate, how much will he earn in 18 days?

26. If $12\frac{1}{2}$ bushels of oats cost $ 5, what will $6\frac{1}{4}$ bushels cost at the same rate?

27. The dividend is 165, the quotient is $6\frac{7}{8}$. What is the divisor?

28. If $5\frac{1}{2}$ tons cost $ 28.27, what will $12\frac{3}{5}$ tons cost?

29. A contractor employs 6 men for $27\frac{1}{2}$ days and pays them $ 363. Find the average daily wages for each man.

30. A merchant buys potatoes at 60¢ a bushel and sells them at 20¢ a peck. Find his profit on 5 bushels.

31. How many bushels of potatoes, at 65¢ each, will pay for 65 yd. of muslin at 15¢ a yard?

32. Mr. Smith bought a farm for $5000. He paid $2500 cash, and is to pay the balance in 4 yearly payments. How much must he pay each year?

33. The value of 9 horses and 8 mules is $3244. If the value of each horse is $196, what is the value of each mule?

34. How many yards of gingham at $8\frac{3}{4}$¢ a yard can be bought for $4?

35. If $\frac{5}{7}$ of a mill is worth $8795, what is $\frac{3}{8}$ of the mill worth at the same rate?

36. What will 8.875 tons of hay cost at $\$12\frac{3}{4}$ a ton?

37. Find the cost of 36 pieces of calico, each containing $42\frac{1}{2}$ yards, at $6\frac{1}{4}$¢ a yard.

38. At the rate of $5\frac{1}{2}$ barrels for $39.50, what will 11 barrels cost? (How many times as much?)

39. Find the cost of 2580 laths at 80¢ a hundred.

40. If $\frac{5}{8}$ yd. of ribbon cost 25¢, what will $\frac{1}{4}$ yd. cost?

41. A boy bought $\frac{3}{4}$ bushel of chestnuts for $2 and sold them at 10¢ a quart. How much did he gain?

42. The difference between two fractions is $\frac{1}{3}$, and the larger fraction is $\frac{7}{12}$. What is the smaller fraction?

43. If $9\frac{1}{2}$ tons of hay cost $95, how many tons can be bought for $50?

44. An heir gets $\frac{8}{9}$ of an estate and invests $1600, which is $\frac{5}{8}$ of his share. How much was his share? How much was the value of the whole estate?

45. What would a gallon of milk cost at 2¢ a pint?

46. Find the cost of 5 pounds of pepper at 3¢ an ounce.

47. Bought 16 pounds of 60-cent tea. How much change should I receive from a $10 bill?

48. How many weeks will 21 lb. of butter last, if $\frac{1}{2}$ lb. is used each day?

49. The divisor is 75, the quotient 657. What is the dividend?

50. A woman paid $24 for 36 yards of dress goods. What did she pay per yard?

51. How many days will 36 bushels of oats last 12 horses, if each horse eats 6 quarts each day?

52. The yearly rent of a house is $480. What is the rent for 2 yr. 4 months?

53. What will 3 bushels of oats cost at 12¢ a peck?

54. At 9¢ a quart, what is the cost of 2½ gallons of vinegar?

55. How many times is 4½ contained in 27?

56. Divide 8765,434 by 902.

57. A boy hires a boat at 20¢ an hour. How much has he to pay if he uses it from 30 minutes past 9 until noon?

58. If ⅔ yard of cloth cost 12¢, how much cloth can be bought for 40¢?

59. Find the cost of 38,640 lb. of wheat at 95¢ a bushel. (60 lb. to the bushel.)

60. Find the cost of 33,600 lb. of corn at 48½¢ a bushel. (56 lb. to the bushel.)

61. Multiply 693.6 by 785.09 and then divide the product by 25.

62. Bought 5 bushels of berries for $5.00 and sold them at 20¢ a quart. How much did I gain?

63. Add ½ to ¾ and then subtract the sum from 5.

64. If .625 of a cord of wood costs $3, what will .75 of a cord cost?

65. If ⅝ of a cord of wood costs $3.00, what will ¾ of a cord cost?

CHAPTER III

FACTORS AND MULTIPLES

122. 1. The integers which multiplied together produce a number are called **factors** of that number. 3 and 7 are factors of 21.

We use factors in changing fractions to their lowest terms, as we then reject equal factors from both numerator and denominator. Factors which are common to two or more numbers are often called **common divisors**.

2. A prime number is one that has no exact divisors except itself and 1. 1, 3, 5, 7, 11, 13, 17, 19, etc., are prime numbers.

3. An even number is one that is exactly divisible by 2. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, etc., are even numbers.

4. All numbers not exactly divisible by 2 are called odd numbers. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, etc., are odd numbers.

Is a prime number and an odd number the same thing?

5. The exact divisors of a number are found by trial, but it will help you in changing fractions to

lowest terms to remember that any number is divisible by

2 if its right-hand figure is 0, 2, 4, 6 or 8.
3 if the sum of its digits is divisible by 3.
5 if its right-hand figure is 0 or 5.
9 if the sum of its digits is divisible by 9.

6. The smallest number that is exactly divisible by two or more numbers is called their **least common multiple.**

We find the least common multiple of the denominators of fractions when we wish to find the new denominator for use in changing them to similar fractions.

7. In adding $\frac{1}{2}$ and $\frac{1}{3}$ we at once see that 6 is the least common multiple of the denominators, and change the fractions to 6ths.

In adding large fractions, like $\frac{3}{64}$ and $\frac{39}{320}$, we cannot tell at once what is the least common denominator, so it is well for us to learn how to find it.

Finding the Least Common Multiple.

123. 1. To add $\frac{3}{64}$ and $\frac{39}{320}$, we use the following form:

$$\begin{array}{l} \frac{3}{64} = \frac{15}{320} \\ \frac{39}{320} = \frac{39}{320} \\ \qquad \frac{54}{320} \textit{ Ans.} \end{array} \qquad \begin{array}{r|rr} 2 & 64 & 320 \\ \hline 2 & 32 & 160 \\ \hline 2 & 16 & 80 \\ \hline 8 & 8 & 40 \\ \hline & 1 & 5 \end{array} \qquad \begin{array}{r} 5 \\ 64\overline{)320} \\ 320 \\ \hline 3 \times 5 = 15 \\ \overline{64} \times 5 = \overline{320} \end{array}$$

$$2 \times 2 \times 2 \times 8 \times 5 = 320 \text{ L.C.M.}$$

EXPLANATION:

We arrange the denominators in convenient form, and divide them by common factors until there remains no common factor.

We then multiply these common factors and the remaining undivided numbers together, and the product is the least common multiple of the numbers, which is the least common denominator of the fractions. We now know that we must change the fractions to 320ths. To see what to multiply the terms of the fraction $\frac{3}{64}$ by to change it to 320ths, we find how many times 64 is contained in 320.

While the fraction $\frac{54}{320}$ written above is the answer, it can yet be changed to lower terms, as 2 will divide both numerator and denominator, and the answer in its simplest form will be $\frac{17}{160}$.

ADDITION OF FRACTIONS

124. Involving the use of least common multiple.

1. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{12}$, and $\frac{9}{16}$.

$$\begin{array}{r} \frac{2}{3}=\frac{32}{48} \\ \frac{3}{4}=\frac{?}{48} \\ \frac{7}{12}=\frac{?}{48} \\ \frac{9}{16}=\frac{?}{48} \\ \frac{?}{48} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 12 & 16 \\ \hline 2 & 3 & 2 & 6 & 8 \\ \hline 3 & 3 & 1 & 3 & 4 \\ \hline & 1 & 1 & 1 & 4 \end{array}$$

$$4 \times 3 \times 2 \times 2 = 48 \text{ L.C.M.}$$

$$\begin{array}{r} 16 \\ 3\overline{)48} \\ 12 \\ 4\overline{)48} \\ 4 \\ 12\overline{)48} \\ 3 \\ 16\overline{)48} \end{array}$$

The full work is shown above, although you will not need to write out your work in full each time, as you can do much of it mentally. To complete the work in the above example, change the remaining fractions to 48's, then add the numerators, and change the sum to its simplest form.

2. Add $\frac{5}{7}$, $\frac{3}{14}$, and $\frac{4}{21}$.

3. Add $18\frac{5}{8}$, $36\frac{5}{12}$, and $108\frac{3}{4}$.

(Use the line instead of the = sign, unless you write the entire mixed number on each side of the = sign.)

4. Find the sum of $75\frac{1}{4}$, $9\frac{1}{5}$, $14\frac{1}{10}$, $36\frac{1}{2}$, and $15\frac{3}{5}$.

5. Add $48\frac{5}{12}$, $24\frac{13}{15}$, and $65\frac{7}{20}$.

6. Add $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{3}{4}$, $4\frac{4}{5}$, and $5\frac{5}{6}$.

7. Add $41\frac{1}{2}$, $105\frac{2}{9}$, $300\frac{3}{4}$, $241\frac{3}{5}$, and $472\frac{1}{4}$.

8. Add $36\frac{5}{8}$, $42\frac{2}{3}$, $39\frac{7}{16}$, and $51\frac{1}{4}$.

9. Add $6\frac{3}{5}$, $8\frac{4}{7}$, $12\frac{5}{21}$, and $18\frac{2}{15}$.

10. Add $18\frac{1}{3}$, $24\frac{1}{4}$, $17\frac{1}{12}$, and $5\frac{8}{15}$.

11. A farmer used $77\frac{1}{2}$ acres of land for wheat, $40\frac{7}{8}$ acres for corn, $1\frac{5}{8}$ acres for vegetables, $29\frac{1}{2}$ acres for pasturage, and $10\frac{1}{4}$ acres for an orchard. How many acres were there in the entire farm?

12. A lady bought three pieces of cloth containing $125\frac{7}{8}$, $96\frac{3}{4}$, and $48\frac{2}{3}$ yd. How many yards in the three pieces?

13. A merchant sold a lady $22\frac{1}{2}$ yd. of silk, $3\frac{1}{4}$ yd. of muslin, $1\frac{1}{8}$ yd. silesia, $5\frac{3}{4}$ yd. cambric, and $5\frac{1}{3}$ yd. ruffling. How many yards were sold?

14. A farmer divides his farm into five fields. The first contained $26\frac{7}{12}$ acres, the second $40\frac{16}{21}$ acres, the third $51\frac{6}{7}$ acres, the fourth $59\frac{3}{4}$ acres, and the fifth $62\frac{2}{3}$ acres. How many acres in the farm?

DENOMINATE NUMBERS

125. In the use of denominate numbers we need to learn to change them from one denomination to another without changing their value, and add, subtract, multiply, and divide them.

If we consider these operations of reduction, addition, subtraction, multiplication, and division with an easy table, and learn how to work with this table, we can use the same plan with all tables which we have time to learn, or need to use.

TABLE FOR DRY MEASURE

2 pints = 1 quart.
8 quarts = 1 peck.
4 pecks = 1 bushel.

Changing to lower denominations. (Reduction descending.)

126. 1. Change 3 bushels, 2 pecks, 3 quarts to quarts.

$$\begin{array}{lllllll} 3\text{ bu.} & & 2\text{ pk.} & & 3\text{ qt.} & = & ?\text{ quarts.} \\ 3 & 4 & \underline{12} & 8 & \underline{112} & & \\ & & 14 & & 115 & & 115\text{ quarts } \textit{Ans.} \end{array}$$

Short Analysis. Think, 4 pk. 1 bu., 3 times 4 or 12 pk. for 3 bu. 12 pk. and 2 pk., 14 pk. 8 qt. in 1 pk., 14 times 8 qt. in 14 pk. 112 quarts and 3 quarts, 115 quarts.

2. Change 5 bu. 1 pk. 6 qt. to quarts.

$$\begin{array}{lllllll} 5\text{ bu.} & & 1\text{ pk.} & & 6\text{ qt.} & = & ?\text{ quarts.} \\ 5 & 4 & \underline{20} & 8 & \underline{168} & & \\ & & 21 & & 174 & & 174\text{ quarts } \textit{Ans.} \end{array}$$

3. Reduce 8 bu. 3 pk. 5 qts. to quarts.

$$\begin{array}{lllllll} 8\text{ bu.} & & 3\text{ pk.} & & 5\text{ qt.} & = & ?\text{ quarts.} \\ 8 & 4 & \underline{32} & 8 & \underline{280} & & \\ & & 35 & & 285 & & 285\text{ quarts } \textit{Ans.} \end{array}$$

4. Reduce 10 bu. 3 pk. 5 qt. to quarts.

5. Reduce 4 bu. 2 pk. 7 qt. to quarts.

6. Reduce 15 bu. 3 pk. 6 qt. to quarts.

7. Reduce 2 bu. 3 pk. 4 qt. 1 pt. to pints.

$$\begin{array}{lllllllll} 2\text{ bu.} & & 3\text{ pk.} & & 4\text{ qt.} & & 1\text{ pt.} & = & ?\text{ pints.} \\ 2 & 4 & \underline{8} & 8 & \underline{88} & 2 & \underline{184} & & \\ & & 11 & & 92 & & 185 & & 185\text{ pints } \textit{Ans.} \end{array}$$

8. How many pints in 3 bu. 3 pk. 3 qt. and 1 pt.?

9. Change 4 bu. 1 pk. 7 qt. and 1 pt. to pints.

10. Change 60 bu. 3 pk. 5 qt. 1 pt. to pints.

11. Reduce 5 bu. 3 pk. 2 qt. to quarts.

12. Reduce 6 bu. 1 pk. 7 qt. to quarts.

13. Reduce 6 bu. 3 pk. 4 qt. 1 pt. to pints.

14. Reduce 8 bu. 3 pk. 6 qt. and 1 pt. to pints.

15. How many pints in 6 bu. 1 pk. 5 qt. and 1 pt.?

Changing to higher denominations. (Reduction ascending.)

127. **1.** Change 115 quarts to higher denominations.

115 qt. = ? bu. ? pk. ? qt.
8)115 qt. 3 qt.
4) 14 pk. 2 pk. 3 bu. 2 pk. 3 qt. *Ans.*
3 bu.

SHORT ANALYSIS. 115 qt. = 14 pk. and 3 qt.
14 pk. = 3 bu. and 2 pk.
115 qt. = 3 bu. 2 pk. 3 qt.

2. Change 174 quarts to higher denominations.

3. Reduce 285 quarts to higher denominations.

4. Reduce 3401 pints to higher denominations.

5. Reduce 460 quarts to higher denominations.

6. Change 348 quarts to higher denominations.

7. Change 145 quarts to higher denominations.

8. How many bushels, pecks, and quarts of oats in 202 quarts of oats?

9. How many bushels of oats in 256 quarts of oats?

10. Reduce 131 quarts to higher denominations.

ADDITION OF DENOMINATE NUMBERS

128. 1. Find the sum of 3 bu. 2 pk. 4 qt.; 4 bu. 1 pk. 3 qt.; 8 bu. 3 pk. 6 qt.

1	1	
3 bu. (4)	2 pk. (8)	4 qt.
4	1	3
8	3	6
	7	13
16 bu.	3 pk.	5 qt. *Ans.*

Explanation. We write the numbers in proper columns, and then add the column of lowest denomination, getting for the sum 13 qt., which is 1 pk. and 5 quarts. We write the 5 and carry the 1 to the top of the next column of units, as we did in adding simple numbers; then we add the column of pk. and get 7 pecks, which is 1 bu. and 3 pk. We write the 3 pk. and carry the 1 bu. to the top of the bushel column, and then add that column. After some practice, you will not need to use the double lines of footing, but can carry, as in simple numbers, thinking 13 qt., 1 pk. and 5 qt., etc.

2. Add the following:

6 bu.	3 pk.	2 qt.
12	2	7
3	3	6
15	2	4

3. Find the sum of 1 bu. 3 pk. 1 qt.; 4 bu. 3 pk. 2 qt.; 8 bu. 3 pk. 0 qt.; 2 pk. 6 qt.; 12 bu. 0 pk. 7 qt.

4. Add the following:

16 bu.	3 pk.	4 qt.	1 pt.
8	2	6	0
	3	0	1
4	0	7	0
	3	2	1

SUBTRACTION OF DENOMINATE NUMBERS

129. 1. From 3 bu. 2 pk. 3 qt. take 1 bu. 2 pk. 4 qt.

3 bu.	(4)	2 pk.	(8)	3 qt.
1		2		4
1 bu.		3 pk.		7 qt. *Ans.*

Short Analysis. Write the numbers in proper columns, then write in the scale or table between the columns, as shown above.

We then subtract as we have learned to do in simple numbers.

In subtracting in simple numbers, the scale or table was always 10, while in denominate numbers the scale is different for different denominations.

In reading the subtraction given above, we read, 4 and 7 = 11; 3 and 3 = 6; 2 and 1 = 3, if we have learned the "additive" plan for subtraction, shown in Chapter I of this book. If we subtract by "borrowing," we "borrow" "8" instead of 10, as we did in simple numbers, as it takes 8 of the units we are using to make 1 of the next higher units, when we are considering quarts. In subtracting pecks and "borrowing" we "borrow" 4 instead of 10, as it takes 4 of these units to make 1 of the next higher units.

The scale written in between the columns of units tells you what to "borrow" each time, if you need to borrow at all. Aside from this, the work is like subtraction of simple numbers.

2. From 18 bu. 2 pk. 7 qt. 1 pt. subtract 4 bu. 3 pk. 4 qt. 0 pt.

3. Subtract,

$$\begin{array}{rrrr} 12 \text{ bu.} & 2 \text{ pk.} & 5 \text{ qt.} & 0 \text{ pt.} \\ -8 & 3 & 7 & 1 \\ \hline \end{array}$$

4. Subtract,

$$\begin{array}{rrr} 34 \text{ bu.} & 1 \text{ pk.} & 3 \text{ qt.} \\ -1 & 3 & 5 \\ \hline \end{array}$$

5. Add,

$$\begin{array}{rrrr} 15 \text{ bu.} & 3 \text{ pk.} & 5 \text{ qt.} & 1 \text{ pt.} \\ 24 & 3 & 6 & 1 \\ \hline \end{array}$$

MULTIPLICATION OF DENOMINATE NUMBERS

130. 1. Multiply 3 bu. 3 pk. 3 qt. by 6.

$$\begin{array}{rrr} 3 \text{ bu.} & (4)\ 3 \text{ pk.} & (8)\ 3 \text{ qt.} \\ & & \times 6 \\ \hline 5 & 18+2 & 18 \\ \hline 23 & 0 \text{ pk.} & 2 \text{ qt.} \end{array} \textit{ Ans.}$$

SHORT ANALYSIS. 6 times 3 qt. = 18 qt., which is 2 pk. and 2 qt; write the 2 qt., carry the 2 pk. to column of pk. 6 times 3 pk. = 18, and the 2 make 20 pk., which is 5 bu. and no pk.; write the cipher, carry the 5 to column of bushels; 6 times 3 bu. are 18 bu., and the 5 bu. make 23 bu.

For convenience, at first, we may use the double lines, as shown above, that we may write down the number carried in its proper place and not have so much to remember. After a little practice, we should carry this number in mind, as we do when we multiply and carry in multiplication of simple numbers.

2.

$$\begin{array}{r} 4\text{ bu. }2\text{ pk. }5\text{ qt. }1\text{ pt.}\\ \times\ 3\\ \hline \end{array}$$

3.

$$\begin{array}{r} 3\text{ bu. }3\text{ pk. }3\text{ qt. }1\text{ pt.}\\ \times\ 7\\ \hline \end{array}$$

DIVISION OF DENOMINATE NUMBERS

131. 1. Divide 11 bu. 3 pk. 7 qt. by 4.

$$\begin{array}{rcccc} 2\text{ bu.} & & 3\text{ pk.} & & 7\text{ qt. }1\tfrac{1}{2}\text{ pt.}\\ \hline 4)11\text{ bu.} & (4) & 3\text{ pk.} & (8) & 7\text{ qt.} \end{array}$$

Short Analysis. 11 bu. is 2 4's and 3 bu.; 3 bu. 3 pk. is 15 pk.; 15 pk. is 3 4's and 3 pk.; 3 pk. and 7 qt. is 31 qt.; 31 qt. is 7 4's and 3 qt.; 3 qt. is 6 pt.; 6 pt. is 1½ 4's.

If the work of carrying and changing is too much to remember at first, you may write it out, as we have to do in simple numbers when we have a large divisor.

$$\begin{array}{rccccc} 2\text{ bu.} & & 3\text{ pk.} & & 7\text{ qt.} & 1\tfrac{1}{2}\text{ pt.}\\ \hline 4)11\text{ bu.} & (4) & 3\text{ pk.} & (8) & 7\text{ qt.} & 0\text{ pt.}\\ & & \underline{12} & & \underline{24} & \underline{6}\\ & & 15 & & 31 & 6 \end{array}$$

2. Divide 14 bu. 2 pk. 5 qt. 1 pt. by 5.
3. Divide 10 bu. 1 pk. 6 qt. 0 pt. by 6.
4. Divide 15 bu. 3 pk. 7 qt. 1 pt. by 8.
5. Divide 7 bu. 2 pk. 3 qt. 0 pt. by 3.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF DENOMINATE NUMBERS, USING TABLE FOR DRY MEASURE

132. 1. Add,

$$\begin{array}{rrrr} 5\text{ bu.} & 3\text{ pk.} & 5\text{ qt.} & 1\text{ pt.} \\ 3 & 1 & 7 & 0 \\ & 2 & 5 & 1 \\ 5 & 1 & 6 & 0 \\ 13 & 3 & 4 & 1 \\ \hline \end{array}$$

2. Subtract,

$$\begin{array}{rrrr} 12\text{ bu.} & 3\text{ pk.} & 3\text{ qt.} & 0\text{ pt.} \\ -8 & 3 & 5 & 1 \\ \hline \end{array}$$

3. Multiply,

$$\begin{array}{rrrr} 4\text{ bu.} & 3\text{ pk.} & 5\text{ qt.} & 1\text{ pt.} \\ & & & \times 5 \\ \hline \end{array}$$

4. Divide, $4\overline{)13\text{ bu. }3\text{ pk. }5\text{ qt. }1\text{ pt.}}$

Write out the analysis for each of the above examples, and be sure that you understand the plan well, so that we may take up new tables, and use the plan for this table without any trouble.

133. COMMON TABLES

Note. The tables are printed together on the following pages for convenience in reference. It is not necessary that all be learned before the applications are begun.

TABLE OF LENGTH

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
1760 yd. or 5280 ft. = 1 mile (mi.)

In some sections of the country the rod of $5\frac{1}{2}$ yd. or $16\frac{1}{2}$ ft. is still used for measuring distances. Sailors use the fathom of 6 feet for measuring depths of water.

(Tables for square and cubic measures will be given when those forms of measurement are taken.)

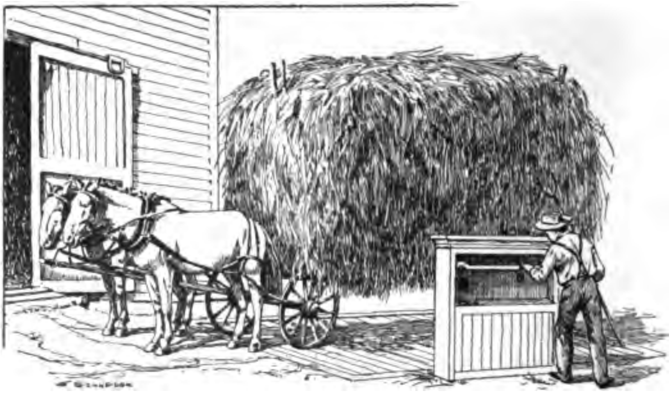


TABLE OF WEIGHT

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

TABLE OF LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Barrels vary in size, but in estimating the contents of tanks it is customary to consider the barrel of 31.5 gallons.

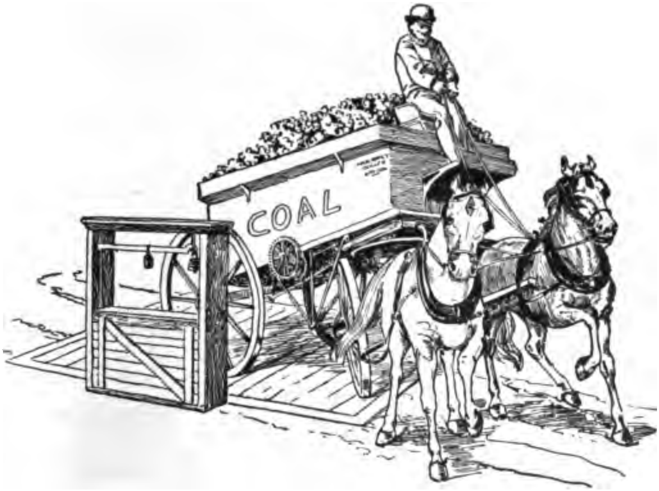


TABLE OF DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

The dry quart is a little larger than the liquid quart.

TABLE OF TIME

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months = 1 year (yr.)

"Thirty days hath September,
April, June, and November."

The following tables are sometimes used:

COUNTING

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

PAPER

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

134. Changing from one denomination to another. (Reduction.)

1. Express 3 qt. 1 pt. as pints. As quarts.
2. How many inches in 3 yd. 2 ft. 6 in.?
3. How many pints in 1 gal. 3 qt. 1 pt.?
4. How many minutes in 1 day 5 hr. 45 min.?
5. How many sheets of paper in one ream?
6. Change 23 pints to pints, quarts, and gallons.
7. Change 131 inches to higher denominations.
8. Change 1 bushel to pints.
9. Change 1 gallon to pints.
10. Change 3 qt. 1 pt. to pecks.

3 qt. 1 pt. = 3.5 qt.

$$\begin{array}{r} .4375 \\ 8\overline{)3.5000} \end{array}$$

3.5 qt. = ? pecks.

.4375 peck *Ans.*

11. Express 2 pk. 5 qt. as bushels.

2 pk. 5 qt. = ? quarts. 2 pk. 5 qt. = 21 qt.
—— qt. = ? bushels.
.65625 bu. *Ans.*

1 bu. = 32 qt.
? bu. = 21 qt.

$$\begin{array}{r} .65625 \\ 32\overline{)21.00000} \\ \underline{19\ 2} \\ 1\ 80 \\ \underline{1\ 60} \\ 200 \\ \underline{192} \\ 80 \\ \underline{64} \\ 160 \\ \underline{160} \end{array}$$

This long operation is shown, not to suggest the use of many exercises like this, but that the pupils, should need arise, may refer to this work as a guide.

Short Analysis. We change the pecks and quarts to quarts. As there are 32 quarts in a bushel, the 21 quarts may be expressed as $\frac{21}{32}$ bushel. This has been changed to the decimal form, and the answer given in decimal form.

$\frac{21}{32}$ or .65625 bushel *Ans.*

12. At 35¢ a quart, what will 3 qt. 1 pt. strawberries cost?

13. At 50¢ a bushel, what will 5 bu. 3 pk. oats cost?

14. At 30¢ a yard, what will 6 yd. 2 ft. ribbon cost?

15. At 5¢ a pint, what will 2 gallons of cider cost?

16. At 15¢ a peck, what will $3\frac{1}{2}$ bushels of apples cost?

17. What will .375 bushel cost at 90¢ a bushel? At 12¢ a peck?

18. At 4 cents a quart, what is the cost of a can of milk which holds 2 gallons and 1 pint?

19. What is the cost of 1 bu. 3 pk. 2 qt. of berries at 12¢ a quart?

20. What will 8 lb. 6 oz. of lard cost at 10¢ a pound?

21. How many quart boxes will 1 bu. 2 pk. 5 qt. of berries fill?

22. If a man averages $2\frac{1}{2}$ feet for each step taken, how many steps will he take in walking $\frac{1}{8}$ mile?

23. What will 4 gallons of sirup cost at 25¢ a pint?

135. DENOMINATE FRACTIONS

Comparatively little time should be given to the consideration of denominate fractions, as their use is very rare outside of the drill in the class room, or on tests for scholarship.

1. What part of a gallon is 2 qt. 1 pt.?

1 gal. $= (4 \times 2)$ 8 pints.

2 qt. 1 pt. $=$ 5 pints. $\frac{5}{8}$ or .625 *Ans.*

2. Change $\frac{5}{8}$ gallon to lower denominations.

$\frac{5}{8}$ gal. $= 4 \times \frac{5}{8}$ qt. or $\frac{5}{2}$ qt.

$\frac{5}{2}$ qt. $= 2\frac{1}{2}$ qt.

$\frac{1}{2}$ qt. $= 2 \times \frac{1}{2}$ pt. or 1 pt. 2 qt. 1 pt. *Ans.*

3. Reduce .885 yd. to feet and inches.

.885 yd. = 3 × .885 ft. or 2.655 ft.
.655 ft. = 12 × .655 in. or 7.86 in.
2 ft. 7.86 in. *Ans.*

4. What part of a bushel is 3 pk. 2 qt. 1 pt.?

5. Find the cost of 2 pk. 5 qt. of —— at 75¢ a bushel.

6. Find the cost of 2 qt. 1 pt. vinegar at 36¢ a gallon.

7. What decimal part of a mile is 220 yd.?

8. What will it cost to build 110 yd. of wire fence, if $\frac{1}{2}$ mile of the same fence costs $250?

9. At the rate of 35¢ a gallon, what will 3 qt. 1 pt. cost?

10. At the rate of $1.50 a bushel, what will 2 pk. 5 qt. cost?

11. What fractional part of a bushel is $\frac{3}{5}$ peck?

$\frac{3}{5}$ pk. = $\frac{1}{4}$ of $\frac{3}{5}$ bu. or $\frac{3}{20}$ bu. $\frac{3}{20}$ bu. *Ans.*

12. What decimal part of a mile is 500 yd.?

500 yd. = $\frac{500}{1760}$ mile. $\frac{500}{1760}$ = ? decimal.

13. Change .007 bushel to pints.

.007 bu. = (.007 × 4 × 8 × 2) pints.

14. Reduce $\frac{8}{9}$ gill to fraction of a gallon.

$\frac{8}{9}$ gill = $(\frac{8}{9} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4})$ gallons.

15. Express $\frac{3}{8}$ mile in yards.

16. Express .0125 bu. in pints.

17. Express 564 minutes in higher denominations.

18. Express $\frac{4}{25}$ ton in pounds.

19. Express 675 lb. in tons.

20. Express 475 inches in higher denominations.

21. Express 4.75 yd. in other denominations.

DENOMINATE NUMBERS

136. Addition and subtraction application applied to various tables.

1. Add:

5 mi.	450 yd.	2 ft.	9 in.
3	75	1	6
	500	2	1
7	800	2	7
2	750	1	6

2. Subtract:

12 gal.	3 qt.	0 pt.
6	3	1

3. Subtract:

15 mi.	0 yd.	0 ft.	0 in.
5	100	2	7

4. From $\frac{1}{2}$ bushel subtract 3 quarts. Give answer in quarts.

5. Find the sum of $\frac{1}{4}$ bu. and 3 pk. 7 qt.

6. Find the difference between $\frac{1}{8}$ mile and 500 yards.

7. Find what decimal part of a mile is equal to 660 yd.

8. From $4\frac{4}{5}$ yards subtract 1 ft. 6 in.

9. Find the sum of $\frac{1}{8}$ gal. and 2 qt. 1 pt.

10. If $\frac{3}{4}$ bushel costs 60 ¢, how many quarts can you buy for 40 ¢?

11. Find the sum of 14 gal. 3 qt. 1 pt. 3 gi.

12. How long was it from May 14, 1865, to Dec. 7, 1890?

$$\begin{array}{rrr} 1890\ (12) & 12\ (30) & 7 \\ 1865 & 5 & 14 \\ \hline 25 & 6 & 23 \end{array} \quad \textit{Ans.}$$

December is the 12th month.
May is the 5th month.

13. How old was a man on Jan. 7, 1909, who was born April 7, 1861?

14. A man loaned some money on June 6, 1908, and it was paid to him on August 12, 1909. How long was the money loaned?

15. What is the length of time from July 4, 1904, to the present time?

16. Find the difference of time between Oct. 12, 1492, and July 4, 1776.

17. Find the difference of time between April 3, 1885, and May 7, 1837.

18. On July 6, 1909, a boy was 8 yr. 3 mo. 12 days old. What was the date of his birth?

$$\begin{array}{rrr} 1909\ (12) & 7\ (30) & 6 \\ -8 & 3 & 12 \\ \hline 1901 & 3 & 24 \end{array} \quad \text{March 24, 1901} \ \textit{Ans.}$$

19. On August 12, 1908, a boy was 14 yr. 9 mo. 24 days old. Find the date of his birth.

20. Find the difference in the dates of the birth of Lincoln and Washington.

21. Lincoln was born Feb. 12, 1809, and became President March 4, 1861. How old was he at that time?

22. Washington was born Feb. 22, 1732, and became President April 30, 1789. How old was he at that time?

137. Multiplication and division applied to various tables.

1. Multiply 4 gal. 2 qt. 1 pt. 3 gi. by 9.

$$\begin{array}{lllll} 4 \text{ gal. } (4) & 2 \text{ qt. } (2) & 1 \text{ pt. } (4) & 3 \text{ gi.} & \\ & & & \times 9 & \\ \hline 42\cancel{6} & 25\cancel{7} & 15\cancel{6} & 27 & \\ \hline 42 \text{ gal.} & 1 \text{ qt.} & 1 \text{ pt} & 3 \text{ gi.} & \textit{Ans.} \end{array}$$

2. A farmer sold 10 loads of hay, each weighing 1 ton 750 lb. How much did he sell?

3. If a farmer raises 80 bu. 2 pk. 6 qt. on each acre, how much will he raise on 9 acres?

4. Multiply 3 yd. 2 ft. 2 in. by 7.

5. If the average weight of 12 bags of potatoes is 130 lb. 12 oz., what is the total weight?

6. Divide 34 gal. 1 qt. 1 pt. by 5.

6 gal. 3 qt. 1 pt. *Ans.*
5)34 gal. (4) 1 qt. (2) 1 pt.

7. Divide 27 days 4 hours by 5.

8. Divide 5 yd. 2 ft. 6 in. by 6.

9. Multiply 8 lb. 10 oz. by 15.

10. Multiply 5 gal. 1 qt. 3 pt. 2 gi. by 6.

11. Multiply 3 hr. 20 min. 35 sec. by 5.

12. Divide 21 gal. 3 qt. 1 pt. 3 gi. by 6.

13. Divide 13 yd. 2 ft. 6 in. by 6.

14. How much wheat can be put in 8 sacks, if each sack holds 2 bu. 3 pk. 6 qt.?

15. If a hogshead of sugar contains 524 lb. 4 oz., what will 8 hogsheads of sugar be worth at 5¢ a pound?

16. What will 5 reams 15 quires of paper cost at 90¢ a ream?

138. MEASUREMENT OF FLAT SURFACES

TABLE FOR AREAS

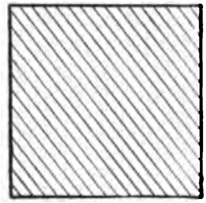
144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)

(The last two units named are used in measuring land areas.)

Note. Care should be used not to confuse this table with the table for measuring lengths. We cannot measure lengths with square units, nor surface with linear units. We cannot say the pencil is 5 square inches long, nor should we say the area of the floor of a room is 40 feet.

Linear Inch

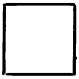
Square Inch



MEASUREMENT OF RECTANGULAR SURFACES

139. **1.** A rectangular surface has square corners.

Square



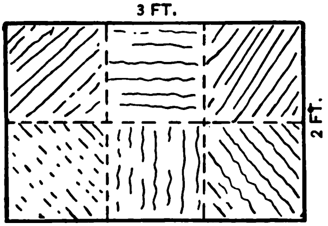
Rectangle

Equal sides.
Square corners.

Parallel sides.
Square corners.

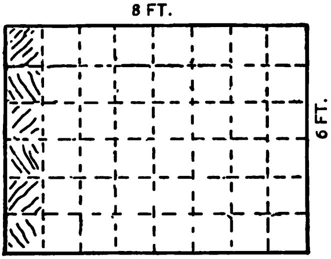
Name some rectangular surfaces. The side of the room is one, the floor is another.

2. How many square feet in the surface of a rectangle 3 feet long and 2 feet wide?



Note that the length and width are measured by linear feet, and the area by square feet.

3. How many square feet in the surface of the floor of a room 8 ft. long and 6 ft. wide?



6 sq. ft. × 8 = 48 sq. ft.

6 square feet measures the surface for 1 tier across the end. The total surface will contain 8 such tiers or 8 times 6 square feet, which is 48 square feet. *Ans.*

4. How many square feet of cement will it take for a cellar floor which is 24 ft. long and 10 ft. wide?

(10 sq. ft. to a tier, 24 tiers.) 10 sq. ft. × 24 = 240 sq. ft. *Ans.*

5. Find the area in square feet of a garden with four sides and square corners, which is 12 yards long and 8 yards wide.

(8 sq. yd. to a tier, 12 tiers.) 8 sq. yd. × 12 = 96 sq. yd. 96 sq. yd. = 9 times 96 sq. ft. or 864 sq. ft. *Ans.*

6. Find the area of a rectangular garden 36 ft. long and 24 ft. wide. Give answer in square feet.

7. How many square yards of carpet will it take to cover a floor which is 18 ft. long and 12 ft. wide?

8. How many square yards of carpet will it take to cover a floor which is 6 yd. long and 4 yd. wide?

140. AREAS OF SQUARES AND RECTANGLES

1. Find the cost at 15¢ a square foot of a cement sidewalk 80 ft. long and 5 ft. wide.

2. How many acres are there in a rectangular field 60 rd. wide and 80 rd. long?

3. How much will it cost to carpet a room with carpet costing $1.25 per square yard if the room is 20 ft. long and 18 ft. wide?

4. A tablecloth is 10 ft. long and 9 ft. wide. How many square yards does it contain?

5. How many square yards of oilcloth will be needed to cover a floor which is 12 ft. square?

6. A city lot is 20 ft. wide and 100 ft. long. How many square feet are there in its area?

7. A rug is 9 ft. by 12 ft. How many square feet in it?

8. A field is 40 rd. long and 40 rd. wide. What is it worth at $75 an acre?

9. How many acres are there in a rectangular field of oats 160 rd. long and 75 rd. wide?

10. How many square inches in the surface of a blackboard which is 40 in. long and 24 in. wide? How many square feet in its surface?

11. Measure the dimensions of a crayon box and find out how many square inches in its entire outer surface.

12. How many square yards in the surface of the side walls and ceiling of a room which is 15 ft. long, 12 ft. wide, and 8 ft. high?

15 sq. ft. × 12 = 180 sq. ft., ceiling.
(15 ft. + 12 ft.) × 2 = 54 ft., distance around the room.
54 sq. ft. × 8 = 432 sq. ft., area of the side walls.
432 + 180 sq. ft. = 612 sq. ft., area of side walls and ceiling.
612 sq. ft. = 68 sq. yd. *Ans.*

13. How many square yards in the side walls and ceiling of a room 12 ft. by 9 ft. and 8 ft. high?

14. How many square feet in the side walls of a room 16 ft. by 8½ ft. and 8 ft. high?

15. How many square feet in the ceiling of a room 18 ft. by 12 ft.?

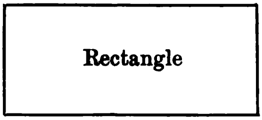
16. How many square yards of carpet will it take to cover the floor of a room 12 ft. by 18 ft.?

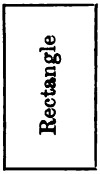
17. How many square feet of zinc will it take to line the sides and bottom of a tank which is 18 in. by 20 in. and 8 in. deep? (Think of a room bottom side up.)

141. AREAS OF PARALLELOGRAMS AND TRIANGLES

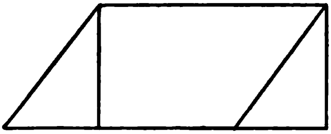
1. A parallelogram is a surface having four sides, its opposite sides being parallel.

2. A rectangle is a special kind of a parallelogram as it has square corners.





3. The area of a parallelogram is equal to that of a rectangle having the same base and altitude.



(The term altitude as here used means the vertical height.)

4. Find the area of the following:

(1) Rectangle, base 12 ft., altitude 6 ft.
12 sq. ft. × 6 = 72 sq. ft. *Ans.*

(2) Parallelogram, base 14 ft., altitude 6 ft.
14 sq. ft. × 6 = 84 sq. ft. *Ans.*

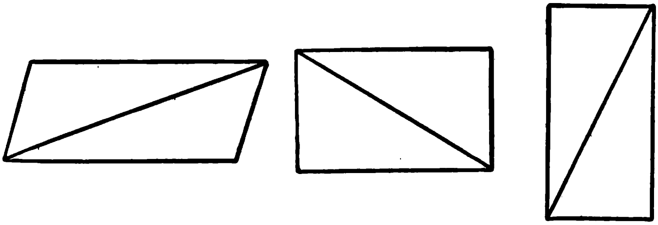
(3) Parallelogram, base 8 in., altitude 6 in.

5. How many square yards in a parallelogram whose base is 12 ft. and altitude 3 ft.?

6. How many square inches in a parallelogram whose base is 16 in. and altitude 12 in.?

MEASUREMENT OF TRIANGLES

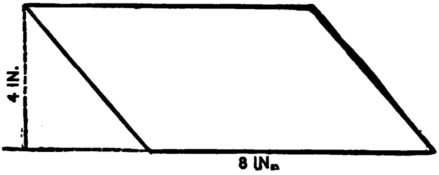
Cut pieces of paper into the form of a rectangle or any kind of a parallelogram. Then cut across the diagonal, thus dividing the parallelogram into two parts. Compare these parts and see if they are equal.



The area of a triangle is ½ that of a parallelogram of the same base and altitude.

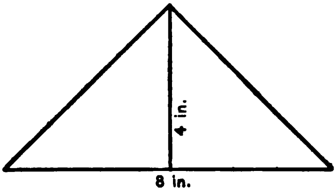
142. 1. Find the area of a parallelogram whose base is 8 in. and whose altitude is 4 in.

8 sq. in. × 4 = 32 sq. in. *Ans.*



2. Find the area of a triangle, base 8 in., altitude 4 in.

$$\frac{8 \text{ sq. in.} \times 4}{2} = 16 \text{ sq. in. } \textit{Ans.}$$

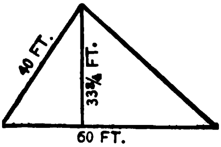


3. What is the area of a triangle, base 10 in., altitude 8 in.?

4. What is the area in square feet of a triangle whose base is 2 yd. and its altitude 5 ft.? (Change base to feet.)

5. What is the area in square inches of a triangle, base 3 ft., altitude 10 in.? (Change base to inches.)

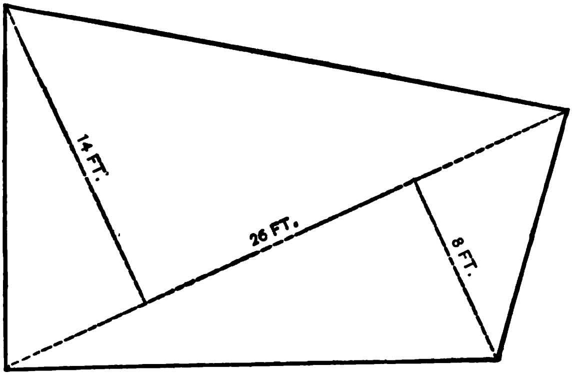
6. A triangular field has the following dimensions, shown on the sketch. What is its area in square feet?



7. What is the area of a parallelogram in square yards whose base is 60 ft. and whose altitude is 30 ft. 6 in.?

8. How many square yards of surface in a sidewalk which is 200 ft. long and 5 ft. 6 in. wide?

9. A garden with four sides is not in the form of a parallelogram, so I made a sketch of it and took the following measurements, as shown on the sketch. Find the area of the garden.



143. FINDING PERIMETER (BOUNDING LINE)

1. Find the perimeter of a garden which is 40 ft. long and 20 ft. wide.

(40 ft. + 20 ft.) × 2 = 120 ft. *Ans.*

2. How many feet of picture molding will it take to go around a room which is 16 ft. by 18 ft.?

(16 ft. + 18 ft.) × 2 = 68 ft. *Ans.*

3. How many feet of fence will it take to inclose a city lot which is 20 ft. by 100 ft.?

Do we use linear feet or square feet to measure the perimeter?

4. Find the cost of fencing a field which is 20 rd. long and 8 rd. wide with fence costing 50¢ a rod.

5. A city lot, 20 ft. by 100 ft., is fenced on the long sides and across the rear end with cheap fence, costing 25¢ a foot. Across the front a better fence is used, costing $ 1.50 a foot. What was the total cost of fencing the lot?

6. Find the area and the perimeter of a field which is 50 yd. long and 30 yd. wide.

7. Paid $ 50 an acre for the land, and $ 1.00 a rod for fencing it. The field was 80 rd. long and 16 rd. wide. Find the total cost of field and fence.

8. How many feet of wire will it take to inclose a lot which is 200 ft. by 150 ft., if 5 lines of wire are used for the fence?

9. How many yards of fence will it take to fence both sides of a railroad for 6 miles?

10. What is the distance around a base ball "diamond"? (90 ft. from base to base.)

11. How many feet of picture molding will it take to frame a picture 18 in. by 15 in., the molding being 2 in. wide? (Make a sketch and allow for corners.)

12. How many feet of fence does it take to inclose a triangular field whose sides are 18 ft., 12 ft., and 15 ft.? (Make a sketch.)

13. Find the perimeter of a square field which is 28 yd. on a side.

Find the area and perimeter of the following rectangular fields:

14. Length 25 rd. Width 18 rd.

15. Length 4 yd. Width 12 ft.
(Ans. in ft. Ans. in yd.)

16. Length $15\frac{1}{2}$ rd. Width $8\frac{1}{2}$ rd.

17. Length 24 ft. Width 9 ft. 6 in.

18. Length 50 yd. Width 24 ft.

19. Length 150 rd. Width 80 rd.

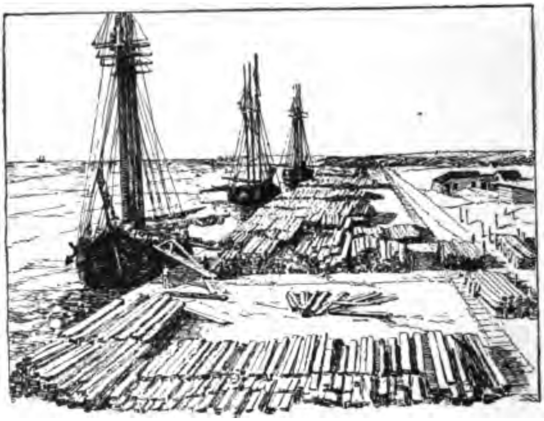
20. Length 320 rd. Width 40 rd.

144. MEASURING LUMBER

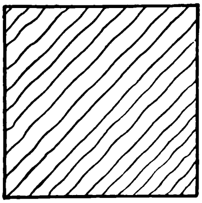
In some sections of the country this measure is little used, and may be omitted.

In measuring lumber boards 1 in. thick, *or less,* are measured by their surface, and the measure called board feet.

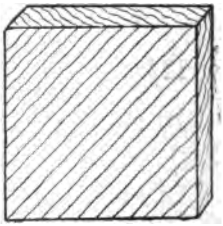
A board foot differs from a square foot, because a square foot has *no* thickness, while a board foot has thickness of 1 inch or less. A board $\frac{1}{2}$ in. thick is measured as if it were 1 in. thick, but a board $1\frac{1}{2}$ in. thick has $1\frac{1}{2}$ times as many board feet as one of the same length and width, which is only 1 in. thick.



Linear foot



Square foot



Board foot

145. 1. How many board feet in a board 1 in. thick, 12 ft. long, and 8 in. wide?

$$12 \text{ sq. ft.} \times \frac{8}{12} = 8 \text{ sq. ft. surface.}$$

As the board is not more than 1 in. thick, it will be of the same number of board feet as its surface measure. 12 bd. ft. *Ans.*

2. How many board feet in a board $\frac{1}{2}$ in. thick, 14 ft. long, and 18 in. wide?

$$14 \text{ bd. ft.} \times \frac{18}{12} = 21 \text{ bd. ft. } \textit{Ans.}$$

(In board measure thickness less than 1 inch is estimated as if 1 inch thick.)

3. How many board feet in a plank 2 in. thick, 18 ft. long, and 8 in. wide?

$$18 \text{ bd. ft.} \times \frac{8}{12} \times 2 = 24 \text{ bd. ft. } \textit{Ans.}$$

(The plank, being 2 inches thick, has twice as many board feet as if it were 1 inch thick.)

4. Find the number of board feet in a timber 30 ft. long, 10 in. wide, and 8 in. thick.

$$30 \text{ bd. ft.} \times \frac{10}{12} \times 8 = ? \text{ bd. ft.}$$

5. Find the number of board feet in a timber 24 ft. long and 9 in. by 8 in.

6. Find the cost at $25 per 1000 bd. ft. of 4 planks each 18 ft. long, 3 in. thick, and 14 in. wide.

7. Find the cost at $22 per 1000 bd. ft. of 4 timbers each 20 ft. long and 8 × 9 in.

8. Find the cost of boards for floor to a room

which is 40 ft. by 15 ft., the boards to be 1 in. thick, and costing $ 25 per 1000 bd. ft.

9. Find the cost at $ 40 per 1000 bd. ft. of a stick of timber 36 ft. long, 12 in. wide, and 9 in. thick.

10. How many board feet of 1-inch lumber will it take to build a tight board fence across the front of a city lot which is 40 ft. front, if the fence is to be 8 ft. high?

146. CUBIC MEASURE

Measurement of volumes and capacities of rectangular solids.

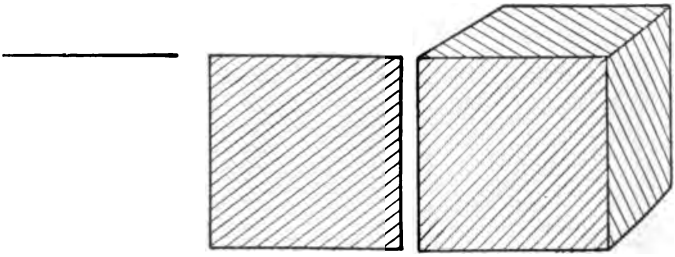
TABLE OF MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

The following equivalents are sometimes used:

231 cu. in. = 1 gallon
2150.42 cu. in. = 1 bushel
128 cu. ft. = 1 cord of wood

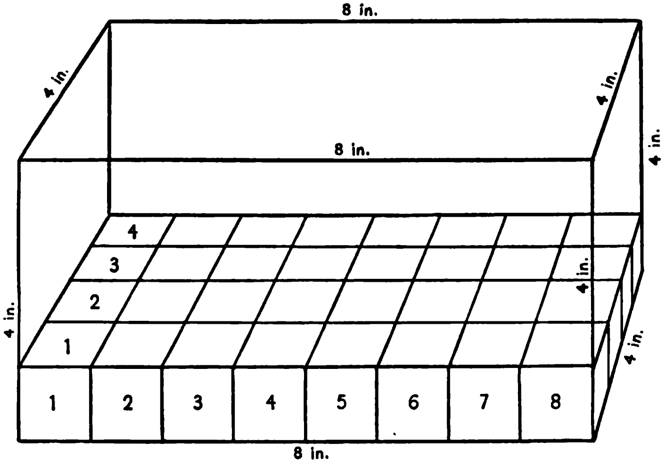
COMPARISON OF UNITS OF MEASURE



Linear inch Square inch Cubic inch

1. How many cubic inches of space in a box whose inside dimensions are 4 in. by 8 in. by 4 in.?

8 cu. in. × 4 × 4 = 128 cu. in. *Ans.*



Think how many 1-inch cubes you could put into the box. You could make 4 rows of 8 cubes in a row for the bottom layer, and could put in 4 such layers; therefore, 8 cu. in. × 4 × 4 = the number of cubic inches of space in the box.

2. How many cubic inches in a rectangular block of wood 12 in. by 6 in. by 4 in.?

Think of the unit of measure again, and how many cubic inches in a row, how many rows across the end for each inch of length, then how many in all.

12 cu. in. × 6 × 4 = 288 cu. in. *Ans.*

3. How many cubic yards of earth will be removed in digging a cellar 30 ft. long, 20 ft. wide, and 5 ft. deep?

30 cu. ft. × 20 × 5 = 3000 cu. ft.
3000 cu. ft. = —— cu. yd. *Ans.*

4. How many gallons of water will a tank 18 in. by 16 in. by 12 in. hold? (231 cu. in. to the gallon.)

18 cu. in. × 16 × 12 = —— cu. in.
—— cu. in. ÷ 231 cu. in. = —— number of gallons *Ans.*

5. How many gallons of water will a tank which is 15 in. by 25 in. by 10 in. hold?

6. How many bushels will a bin 2 ft. by 3 ft. by $1\frac{1}{2}$ ft. hold? (Allow $1\frac{1}{4}$ cu. ft. to the bushel.)

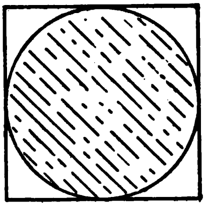
7. How many bushels will a bin hold which is 8 ft. long, 4 ft. wide, and 3 ft. deep?

8. How many gallons will a tank hold which is 2 ft. long, 18 in. wide, and 12 in. deep?

147. MEASUREMENT OF CIRCLES (Introduction)

Only the very easy applications of measurement of circles can be used at this time, as the subject belongs to more advanced grades. It is touched upon here to enable you to work a class of problems which you might need to know how to do before you reach the subject in the higher book.

Study the figure shown below, and you will see that it is not as far around a circle as it is around a square which is the same distance across. You will also see that the area of the circle is less than that of the square.



The following statements you may not understand, but if you will remember the decimal given, you can do many examples in circular measurement by comparing it with square measurement.

Perimeter (circumference) of circle
= .7854 of perimeter of square of same diameter.

Area of circle = .7854 of the area of square of same diameter.

1. What is the area of a circle which is 6 ft. across?

6 sq. ft. × 6 = 36 sq. ft., area of square 6 ft. across.

36 sq. ft. × .7854 = 28.2744 sq. ft. *Ans.*

2. What is the area of a circle which is 10 ft. in diameter?

10 sq. ft. × 10 = 100 sq. ft., area of square 10 ft. in diameter.

100 sq. ft. × .7854 = 78.54 sq. ft. *Ans.*

3. What is the perimeter or circumference of a circle which is 4 ft. in diameter?

4 ft. × 4 = 16 ft., perimeter of square 4 ft. in diameter.

16 ft. × .7854 = ——. *Ans.*

4. Find the area and the circumference of a circle whose diameter is 8 ft. (Calculate first for a square, then use the decimal .7854 to change to circle.)

5. How many cubic inches of water will a milk can hold, if its bottom is circular and 14 in. across, and the can is 30 in. deep?

If the bottom were square, it would hold 14 cu. in. × 14 × 30 cu. in. The relation of the circle to the square is such that with a circular bottom it will hold .7854 the contents of a similar can with a square bottom.

14 cu. in. × 14 × 30 × .7854 *Ans.*

6. How many cubic inches of ashes will an ash can hold, if the can has a circular bottom 15 in. in diameter, and is 32 in. deep?

Compare the volume of the square prism and of the cylinder which you use in drawing, and try to remember the relation number .7854, and also remember that the circle is **less** in perimeter and in area than the square of like dimensions.

148. PROCESSES IN SIMPLE AND IN COMPOUND NUMBERS COMPARED

1. The number 783.45 if written out with the place value names for the figures would be written:

7 hundreds 8 tens 3 units 4 tenths 5 hundredths.

In this form it looks like the form used for denominate numbers:

7 hundreds 8 tens 3 units 4 tenths 5 hundredths (simple).

7 bushels 8 pecks 3 quarts 4 pints (compound number).

ADDITION SIMPLE NUMBERS

hundreds	tens	units	tenths
6 (10)	4 (10)	3 (10)	2
4	3	4	7
8	7	6	5
18	14	13	14
19	5	4	4

Ans. 1954.4.

ADDITION COMPOUND NUMBERS

bushels	pecks	quarts	pints
45 (4)	3 (8)	6 (2)	1
30	3	3	1
12	2	6	1
87	8	15	3
89	2	0	1

Ans. 89 bu. 2 pk. 0 qt. 1 pt.

SUBTRACTION SIMPLE NUMBERS

tens	units	tenths	hundredths
4 (10)	3 (10)	2 (10)	7
1	4	3	9
2	8	8	8

Ans. 28.88.

SUBTRACTION COMPOUND NUMBERS

gallons	quarts	pints
8 (4)	3 (2)	0
2	3	1
5	3	1

Ans. 5 gal. 3 qt. 1 pt.

MULTIPLICATION SIMPLE NUMBERS

hundreds	tens	units	tenths
6 (10)	4 (10)	7 (10)	6
			× 7
42	28	49	42
45	3	3	2

Ans. 4533.2.

MULTIPLICATION COMPOUND NUMBERS

yards	feet	inches
3 (3)	2 (12)	8
		× 7
21	14	56
27	0	8

Ans. 27 yd. 0 ft. 8 in.

DIVISION SIMPLE NUMBERS

tens	units	tenths
0	8	6
5)4 (10)	3 (10)	0

DIVISION COMPOUND NUMBERS

years	months	days
1	7	25
5)8 (12)	3 (30)	5

REDUCTION SIMPLE NUMBERS

hundreds	tens	units
6 (10)	4 (10)	3
6	60	640
	64	643 units

REDUCTION COMPOUND NUMBERS

bushels	pecks	quarts
8 (4)	3 (8)	6
8	32	280
	35	286 qt.

From a careful study of the above, you will see that the main difference between the operations in simple numbers and in compound numbers is in the table of denominations.

149. SUMMARY OF DENOMINATE NUMBERS

1. The operations of reduction, addition, subtraction, multiplication, and division in denominate numbers are like those in simple numbers, except that in simple numbers the scale is always 10, while in denominate numbers it varies with the different units and in different measures.

2. In finding what fractional or decimal part one denominate number is of another, we reduce the numbers to the same denomination and then compare them.

3. In changing denominate fractions or decimals to higher or lower denominations, we multiply or divide the part given by the numbers in the scale of the table of measure used.

4. In finding differences between dates, we write in the number of the month instead of its name, and then proceed as in subtraction of other denominate numbers.

5. In the measurement of flat rectangular surfaces, we find the area by multiplying the number of square units in one dimension by the number of units in the other dimension.

6. A parallelogram is equal to a rectangle of the same base and altitude.

7. A triangle is equal to ½ of a rectangle of the same base and altitude.

8. Perimeter means bounding line or distance around the surface. It is found by adding the length of the sides.

9. Board measure differs from square measure in that it is based upon thickness of one inch or less.

10. In finding the number of board feet, we find the number of surface feet as in square measure, and call them board feet for thickness of one inch

or less. For greater thickness we multiply the board feet of the surface by the number of inches in thickness.

11. Cubic measure deals with three dimensions, length, breadth, and thickness. The unit of measure is a cube.

12. 231 cu. in. of space will hold 1 gallon.

13. 2150.42 cu. in. or $1\frac{1}{4}$ cu. ft. of space will hold 1 bushel.

14. In the surface measure of circles we calculate as if it was a square surface of like dimension, and then take .7854 of the result to change to circular measure.

15. In finding the volume of cylindrical objects, we may calculate as if it were a square prism of like dimensions, and then take .7854 of the result to change to cylindrical measurement.

16. The area of a circle is .7854 of the area of a square which is the same distance across.

17. The circumference of a circle is .7854 of the perimeter of a square of like diameter.

18. The volume of a cylinder is .7854 of the volume of a square prism of like dimensions.

PROBLEMS FOR GENERAL REVIEW

1. Wellington was born May 1, 1769. How old was he at the battle of Waterloo, June 18, 1815?

2. Find the cost of $56\frac{1}{4}$ lb. butter, if 252 lb. cost $84.

3. Find the cost of 45 bu. 3 pk. 1 qt. of wheat at 95¢ a bushel.

4. How many square feet in the surface of a blackboard 22 ft. long and $3\frac{1}{2}$ ft. wide?

5. What will it cost to cement a cellar bottom 42 ft. 6 in. long and 36 ft. wide at 12¢ a square foot?

6. How many acres in a field 160 rd. by $75\frac{1}{2}$ rd.?

7. How many bushels of oats can be put into a bin 10 ft. by 5 ft. by 6 ft.?

8. If $\frac{5}{8}$ lb. cost 50¢, what will $16\frac{3}{4}$ lb. cost?

9. Find the sum of $18\frac{2}{3}$, $7\frac{7}{9}$, $6\frac{5}{21}$.

10. A farm of 157 acres was sold for $4474.50. What was the price per acre?

11. What is the cost of 125 bushels of oats at 48¢ a bushel?

12. At $6.40 a ton, what will 2875 lb. of coal cost? $\$6.40 \times \frac{2875}{2000} = \$$—— *Ans.* (Use cancellation.)

13. Find the cost of 837 lb. of sugar at $5.40 per C.

14. Find the value of 865 lb. of hay at $16 per ton.

15. At $7.35 per M., how much will 12,244 bricks cost?

16. At 35¢ a peck, how many bushels of potatoes can be bought for $117.70?

17. Divide .0752 by .014.

18. Change .4, .25, .875, to fractional form.

19. How many cubic yards of earth must be excavated to make a cellar 48 ft. by 32 ft. by 7 ft.?

20. Write a receipt for rent for last month.

21. Find the difference between 43 and .643.

22. What is the cost of 4868 lb. of sugar at $4.86 a hundred?

23. Find the number of cubic feet of water in a cistern 5 ft. square, if the water is 4 ft. deep in the cistern.

24. Reduce .865 of a gallon to lower denominations.

25. What part of a bushel is 2 pk. 4 qt.?

26. How many quarts are there in .975 of a bushel?

27. If 420 barrels of flour cost $1785, what will 375 barrels of flour cost at the same rate?

28. Reduce 9 gal. 3 qt. 1 pt. 2 gi. to gills.

29. How many quarts are there in 4.35 bushels?

30. Change 2 ft. 8 in. to the decimal of a yard.

31. A boy bought a bushel of chestnuts for $2, and sold them at 5¢ a pint. How much did he gain?

32. At $8.75 a ton, what will 8750 lb. of plaster cost?

33. Find the cost of cementing the sides and bottom of a cistern which is 5 ft. square and 4 ft. deep, at 75¢ a square yard.

34. A floor containing 252 sq. ft. is 14 ft. wide. How long is it?

35. Find the number of acres in a field 60 rd. long and 40 rd. wide. What will it cost to fence the field at 75¢ a rod for fence?

36. A field containing 15 acres is 40 rd. wide. How long is it?

37. Find the cost of 8760 shingles at $ 3.50 per thousand.

38. At $65 an acre, what is the value of a rectangular field 240 rd. long and 88 rd. wide?

39. Find the time between Sept. 28, 1875, and May 12, 1909.

40. How many gallons will a tank hold that is 6 ft. long, 5 ft. wide, and 4 ft. deep?

41. How many board feet in a board that is 18 ft. long, 10 in. wide, and 1 in. thick?

42. How many board feet are there in a stick of timber which is 26 ft. long and 8 in. thick and 10 in. wide?

43. A bin is 10 ft. long and 7 ft. wide. How deep must it be to hold 448 bushels of oats?

44. A farmer sold $\frac{1}{2}$ of his wheat to one man, $\frac{1}{3}$ of it to another, and then had 168 bushels left. How much wheat had he at first?

45. If $\frac{4}{7}$ of a piece of silk is worth $8, what is the value of the other part of the piece at the same rate?

46. The base of a triangle is 6 ft. and its altitude 24 in. What is its area in square feet?

47. If 56 lb. of sugar cost $3.08, what will 24 lb. cost at the same rate?

48. How many feet and inches in $\frac{5}{12}$ yard?

49. From 2700 take 27 hundredths.

50. What decimal of a pound is 14 ounces?

51. How many steps of 2 ft. 6 in. in length must a man take to walk $\frac{1}{8}$ mile?

52. Subtract 3 pecks from $3\frac{7}{8}$ bushels.

53. How many square feet are there in a floor of a room which is 24 ft. long and 18 ft. wide?

54. How many square yards are there in the ceiling of the same room?

55. What decimal of a bushel is 4 quarts?

56. At the rate of $6 a ton, what should be paid for 250 lb. of coal?

57. What will 8 yd. 2 ft. 6 in. of silver wire cost at $8\frac{2}{3}$¢ an inch?

58. Divide 4.50006 by .015.

59. If $\frac{5}{8}$ of a cord of wood cost $3.75, what will .75 of a cord cost at the same rate?

60. What is the area of a circular field which is 100 ft. across?

61. What is the area of a square field which is 100 ft. on a side? What will it cost to fence it at $1.50 a yard?

62. How far is it around a stove pipe which is 6 in. in diameter?

63. How much more land is there in a square field which is 150 yd. across than in a circular field of 150 yd. in diameter?

64. Which will hold the most water, a cistern which is square and 4 ft. deep, or a circular one 4 ft. deep?

65. What is the cost of 18 yd. of cloth at $198 a yard? Can you work this example in a short way?

66. What is the area of a triangle whose base is 140 ft. and its altitude 60 ft.? Give answer in square yards.

67. How many board feet in a plank which is 18 ft. long, 10 in. wide, and 3 in. thick?

68. How many gallons will a tank hold that contains 64 cubic feet?

69. How many bushels will a bin hold which contains 180 cubic feet?

70. A note was given June 6, 1907, and paid Jan. 4, 1908. How long did it run?

71. What is the rent of a house for 7 months at $600 a year?

72. Divide 678,945 by 697.

73. Add the following: $160.75, $67.80, $750, $78.25, $75.20, $350.85, $782.35, $67.30, $75.25, $85.25, $150.75, $65.40.

CHAPTER IV

PERCENTAGE

151. In percentage we shall find many old friends, whom we have learned to know in fractions and decimals under different names.

If half of his crop was destroyed by a storm, a farmer would probably say, "One half of the crop was destroyed," while a business man would say, "Fifty per cent of the crop was destroyed," and in the schoolroom one pupil might say, "Five tenths of the crop was destroyed," and another pupil might say, "Fifty hundredths of the crop was destroyed," and all mean the same thing; as, $\frac{1}{2}$, 50 %, .5, and .50 are only different expressions for the same thing.

Per cent means hundredths. The sign is %.

Study the following table of equal expressions, and see if you can verify it.

100 % = 1.00 = $\frac{100}{100}$ = All.
50 % = .50 = $\frac{1}{2}$ = One half.
25 % = .25 = $\frac{1}{4}$ = One fourth.
20 % = .20 = $\frac{1}{5}$ = One fifth.
$12\frac{1}{2}$ % = .125 = $\frac{1}{8}$ = One eighth.
5 % = .05 = $\frac{1}{20}$ = One twentieth.

Complete the following table of equal expressions:

75 % = .75 = $\frac{3}{4}$
80 % = ——— = ———
40 % = ——— = ———
30 % = ——— = ———
——— = .16 = ———
——— = ——— = $\frac{3}{8}$
125 % = ——— = $1\frac{1}{4}$
60 % = ——— = ———
$\frac{1}{2}$ % = .005 = ———
$\frac{1}{4}$ % = .0025 = ———

152. Application of per cent expression to problems.

1. In a spelling test of 50 words, James spelled 80 % of them correctly. How many words did he spell correctly?

80 % = .80 $50 \times .80 = 40$ 40 *Ans.*

2. A man had $120, and spent 40 % of it for a suit of clothes. How much did the clothes cost?

40 % = .40 $120 × .40 = $48 $48 *Ans.*

3. A man paid $50 for a typewriter, and sold it at a loss of 20 %. What did he get for it?

20 % = $\frac{1}{5}$ $\frac{1}{5}$ of $50 = $10 loss. $50 − $10
= $40 *Ans.*

4. If my income is $3000, and I spend 35 % of it, how much do I save?

35 % = .35 $3000 × .35 = $1050 spent.
$3000 − $1050 = $1950 saved. $1950 *Ans.*

5. Eighty per cent of a class of 45 pupils are promoted. How many are promoted?

$80\,\% = \frac{4}{5}$ $\frac{4}{5}$ of 45 = 36   36 *Ans.*

Note. In computation we use either the fractional or the decimal form of the per cent expression, as is most convenient in that particular exercise. We never use the per cent form in computation.

6. A man buys a house for $5000, and sells it at an advance of 20 %. What does he receive for the house?

One way

$20\,\% = \frac{1}{5}$
$\frac{1}{5}$ of $5000 = $1000 gain.
$5000 + $1000 = $6000 received. $6000 *Ans.*

Another way

100 % of cost = all of cost.
20 % of cost = gain.
120 % of cost = selling price.
$5000 × 1.20 = $6000 selling price. $6000 *Ans.*

7. A farmer had 150 sheep, and sold 20 % of them. How many had he left?

8. A man who received $20 a week had his pay raised 8 %. What were his daily wages then? (6 days.)

9. A barrel of flour weighs 196 lb., and the bread made from it weighs 25% more. How many pounds of bread can be made from 3 barrels of flour?

10. There are 540 pupils in a school, 45% of whom are boys. How many girls are there?

11. A man bought a house for $7500, and sold it at an advance of 24%. How much did he gain? What amount did he receive?

12. My salary last year was $3500, but this year it is 15% more. How much is it this year?

13. Find $12\frac{1}{2}$% of $720.

14. Find $62\frac{1}{2}$% of 800 lb.

15. Find $33\frac{1}{3}$% of 24 bushels.

16. Find 75% of 160 acres of land.

17. A man having 80 sheep sold $\frac{1}{4}$ of them. What per cent of them did he sell?

18. A man having 240 sheep sold 60 sheep. What part of them did he sell? What per cent of them did he sell?

19. In a spelling test of 60 words a boy missed 15 words. What per cent did he spell correctly?

20. I had $50 and spent $5 for a hat. What per cent of my money did I spend? What amount of money did I have left? What part of my money did I have left? What per cent of my money did I have left?

21. From a tank containing 120 gallons of water 30 gallons of water were used. What per cent of the water was used? What amount of water remained in the tank?

22. A boy had 36 marbles and lost 6 of them. What per cent of them did he lose? What per cent of them had he left?

23. A man owned $\frac{4}{5}$ of a farm. What per cent of the farm did he own? If there are 240 acres in the farm, how many acres did he own?

24. A house valued at $4800 was damaged 40 % by fire. What was the amount of the damage?

25. The peanut crop for last year was 40 bushels to the acre, but this year it was 25 per cent more. What was the crop this year on 5 acres of peanuts?

26. The hay crop last year averaged 2 tons to the acre on a meadow of 12 acres. This year it was 25 per cent less. How many tons of hay were raised in the 12-acre meadow this year?

27. The population of a city was 75,000 in 1900. If its growth in population was 10 per cent in 5 years, what was the population in 1905?

INTEREST

A boy hires a boat for 3 hours at 25¢ an hour and pays 75¢, which he calls boat hire. A man borrows the use of a house for 3 months at $25 a month and pays $75, which he calls rent. A man borrows money and pays for its use, and calls the sum paid interest.

Interest is the sum paid for the use of money.

Interest is usually named in terms of per cent of the money borrowed for the time of one year.

Interest at 6 % means that a sum equal to 6 % of the money used will be paid for its use each year.

153. Applications of interest.

1. Find the interest on $350 for 1 year at 6 %.

$\$350 \times .06 = \$21$ interest. $21 *Ans.*

2. Find the interest on $500 for 3 years at 6 %.

$\$500 \times .06 \times 3 = \$$—— *Ans.*

3. Find the interest on $400 for 1 yr. 6 mo. at 6 %.

$\$400 \times .06 \times \frac{18}{12} = \$$—— *Ans.* (18 mo. = $\frac{18}{12}$ yr.)

4. Find the interest on $250 for 5 months at 6 %.

$\$250 \times .06 \times \frac{5}{12} = \$$—— *Ans.*

5. Find the interest on $300 for 2 mo. 12 da. at 6 %.

2 mo. 12 da. = 72 da. 72 da. = $\frac{72}{360}$ yr.

$\$300 \times .06 \times \frac{72}{360} = \$$—— *Ans.*

Note. In interest time is usually reckoned as 360 days to the year.

To arrange the terms so that we may shorten the operations by cancellation, we may write the terms of the fifth example as follows:

$$\frac{\$300}{1} \times \frac{6}{100} \times \frac{72}{360} = \$\text{——}\ \textit{Ans.}$$

6. Find the interest on \$720 for 1 yr. 3 mo. 20 da. at 6%.

1 yr. 3 mo. 20 da. = 470 da. 470 da. = $\frac{470}{360}$ yr.

$$\frac{\$720}{1} \times \frac{6}{100} \times \frac{470}{360} = \$\text{——}\ \textit{Ans.}$$

7. Find the amount due on a note for \$50 which has been at interest for 2 yr. 6 mo. at 6%.

2 yr. 6 mo. = 30 mo. or $\frac{30}{12}$ yr.

$$\frac{\$50}{1} \times \frac{6}{100} \times \frac{30}{12} = \$6.25 \text{ interest.}$$

\$50 + \$6.25 = \$56.25 amount due.

\$56.25 *Ans.*

8. \$500 was borrowed May 1, 1907, and paid Sept. 1, 1909, interest at 6%. What was the amount paid?

$$\begin{array}{rrr} 1909 & 9 & 1 \\ -\ 1907 & 5 & 1 \\ \hline 2 & 4 & 0 \end{array} \quad \text{Time 2 yr. 4 mo. or } \tfrac{28}{12} \text{ yr.}$$

$$\frac{\$500}{1} \times \frac{6}{100} \times \frac{28}{12} = \$70 \text{ interest.}$$

\$500 + \$70 = \$570 amount paid.

\$570 *Ans.*

9. \$200. Brooklyn, N.Y., Jan. 1, 1909.

One year from date, for value received, I promise to pay John Smith, or order, two hundred dollars, with interest at six per cent. James Brown.

Find the amount that Mr. Brown has to pay when the note is due.

$$\$200 \times .06 \times 1 = \$12 \text{ interest.}$$
$$\$200 + \$12 = \$212 \textit{ Ans.}$$

10. What is the interest on $750 for 2 yr. 8 mo. at 5%?

11. Find the amount due on a note for $60, which has been at interest for 9 mo. at 5%.

12. Find the interest on $500 from May 12, 1907, to June 22, 1908, at 6%.

13. Find the amount due on a note for $150, which has been at interest for 1 yr. 4 mo. at 6%.

14. A man loans $5000 at 5%, and arranges to have the interest paid on the first day of January and the first day of July each year during the time the money is loaned. How much interest should he receive at the time of each payment named?

15. Mr. Smith borrows $1500 on Sept. 1, 1907, for 3 yr. at 5%, and agrees to pay the interest each year on Sept. 1. How much is the yearly interest which he has to pay?

16. I sell 500 bu. of oats at 40¢ a bushel, and take a note in payment due in 6 mo., and bearing interest at 6%. How much money should I receive when the note is due?

17. John Adams of Chicago borrows $300 on June 1, 1908, of Frank Brown, and gives him a note for it due in one year, at 6 % interest. How much money should be paid when the note is due?

18. Study the form given on a previous page, and then write the note which Mr. Adams gave to Mr. Brown as outlined in the last example.

A note of this form is often called a promissory note.

Find the interest and the amount of:

19. $ 480 from June 7, 1890, to Dec. 7, 1892, at 5 %.

20. $ 72 from Sept. 10, 1907, to Sept. 25, 1909, at 6 %.

21. $ 360 from April 1, 1906, to July 1, 1908, at 5 %.

22. $ 2500 from Aug. 1, 1908, to Dec. 1, 1909, at 5 %.

23. $ 4500 from Feb. 1, 1906, to Sept. 1, 1909, at 6 %.

24. $ 600 for 2 yr. 4 mo. and 18 da. at 5 %.

25. $1800 for 1 yr. 8 mo. 24 da. at 6 %.

BANK DISCOUNT

Sometimes a man who has money due him needs it before it is due, and will pay something to a person or to a bank, if it will advance the money for him. The sum which they charge for advancing the money is called **discount**. The form which banks have of reckoning the amount of such charge has given it the name of Bank Discount.

Bank discount is the same as interest paid in advance.

The sum which is charged for the advance of the money is called discount.

The balance which is advanced to the man is called the **proceeds**.

154. 1. A man having $ 500 due him in 3 mo., gets a bank to advance it for him, as he needs it at once. The bank charges discount at the rate of 6%. Find the discount and the proceeds.

$$\$500 \times .06 \times \tfrac{3}{12} = \$7.50 \text{ discount.}$$
$$\$500 - \$7.50 = \$492.50 \text{ proceeds.}$$

The bank would pay the man $ 492.50 and then collect the $ 500 when it became due, receiving $ 7.50 for the use of its money during the 3 mo.

2. I have $ 500 of salary which is due me in 2 mo., but need the money at once. How much could I get now, if I sell my claim to the bank for ready money, discount at 6 % ?

3. A grocer sells a debt of $ 240 due in 4 mo. to a bank for ready money, discount at 6 %. How much money does he receive?

4. Which is the better for some goods, and how much better, $ 490 cash, or $ 500 to be paid in 3 mo., if the use of the money is worth 6 % ?

$$\$500 \times .06 \times \tfrac{3}{12} = \$7.50 \text{ discount.}$$
$$\$500 - \$7.50 = \$492.50 \text{ proceeds or cash value.}$$

$ 492.50 – $490 = $2.50, the difference in the two offers. You will gain $ 2.50 by taking the offer to pay $ 500 in 3 mo., as you can sell such a debt to the bank for $ 492.50 cash.

5. Which is worth more, a cash payment of $ 1000, or good security to pay in 4 mo. $ 1050, if money is worth 6 % ?

6. Find the bank discount and the proceeds on $ 240 for 6 mo., the rate of discount being 6 %.

7. Find the bank discount and the proceeds on $ 750 for 4 mo., if the rate of discount is 6 %.

8. A man having a note for $ 50 due him in 4 mo. discounts it at bank at 6 %. How much cash does he receive?

9. Find the bank discount and the proceeds on a note for $ 85 due in 6 mo., discounted at 6 %.

10. Find the bank discount and the proceeds on a note for $ 150 due in 9 mo., discounted at 6 %.

TRADE OR COMMERCIAL DISCOUNT

Often a firm will make a reduction for immediate payment on the goods which it sells, and for which it usually has to wait a short time for payment. This deduction they call discount, and write on their bills, "5 % off for cash," meaning that they will deduct 5 % of the cost if cash is paid, rather than wait for the full payment later.

A firm often prints a general list or catalogue of its goods for sale, and prints the prices for each item. They will then make some discounts for large purchases or to good customers, and this they call commercial discount. They usually write it on the bill "20 % off," or "20 % and 5 % off."

When two or more discounts are quoted, as "20 % and 5 % off," they mean that first 20 % is deducted, and then 5% of the remainder is deducted. 20% and 5% is *not* the same as 25%, as you will see.

155. 1. What will I have to pay in cash for goods listed at $ 60, with 25% off for cash?

$ 60 × .25 = $ 15. Deducted for cash payment.
$ 60 – $ 15 = $ 45. Cash value. $ 45 *Ans.*

2. What will I have to pay for goods listed for $ 60, with 20% and 5% off?

$ 60 × .20 = $ 12, first discount.
$ 60 – $ 12 = $ 48, after first discount.
$ 48 × .05 = $ 2.40, second discount.
$ 48 – $ 2.40 = $ 45.60, after second discount. $ 45.60 *Ans.*

Note. Merchants have a shorter way of finding the cost, but you had better learn the long way thoroughly before you try to use the shorter way.

3. Find the cost of a watch listed at $ 50, with discounts of 25% and 5%.

4. Find the cost of plows listed at $ 15, with discounts of 30% and 10%.

5. Find the cost of goods listed at $ 75, with discount of 10% for cash.

I get a price list from a firm, and they write me a personal letter stating that they will allow me discounts of 25% and 10% on the prices quoted in the list. Find the cost of the following:

6. Watches listed at $ 50.

7. Rings listed at $ 12.

8. Scarf pins listed at $ 5.

9. Bracelets listed at $ 7.50.

A firm sends me its price list, and writes me a letter stating their terms to be "three months, 25 % off for cash." Find the cash cost of the following goods on their list:

10. Wagons listed at $ 80.

11. Plows listed at $ 18.

12. Mowing machines listed at $ 75.

13. Reapers listed at $ 250.

14. Cultivators listed at $ 15.

15. Lawn mowers listed at $ 8.

Note. Their terms mean that I may wait three months and pay full list price, or that I may discount the price 25 % for cash.

COMMISSION

An agent often does business for a man or for a firm, and charges for his work a certain per cent based upon the business done. This charge he calls his commission.

156. 1. Find the commission at 2 % which an agent would charge for selling a house for $ 5000.

$ 5000 × .02 = $ 100 commission *Ans.*

2. Find the agent's commission for selling a house for $ 3000, his rate of commission being $2\frac{1}{2}$ %.

3. Find the commission which an agent would charge for renting a house for $ 300, his rate of commission being 5 %.

4. Find the agent's commission at 5 % for selling 10 tons of hay at $ 20 a ton.

5. At 3 %, what is the commission on the sale of 5000 lb. of sugar at 5½ ¢ a pound?

6. At 3 %, what is the commission on the sale of 20 bbl. of apples at $ 3 per barrel?

7. At 5 %, what is the commission on the sale of 100 doz. of eggs at 30 ¢ a dozen?

8. At 2 %, what is the commission for buying 750 bushels of wheat at 90 ¢ a bushel?

9. An agent sold goods to the amount of $ 1260. What was his commission at 3½ %?

10. An agent is paid 2½ % for buying flour. What is his commission if he buys 300 bbl. at $ 4.50 a barrel?

11. A lawyer charges 5 % commission for collecting a debt of $ 2500. How much does the lawyer charge?

12. An agent charges 2 % commission for buying cheese. How much does he earn in a week, if he buys 8000 lb. at 16 ¢ a pound?

13. A planter pays his agent 5 % commission for selling his cotton. How much does the agent re-

ceive for his commission, if he sells cotton amounting to $ 5000?

14. A commission merchant in New York charges 3 % for selling butter. I send him 150 lb. of butter, which he sells at 25 ¢ a pound. What is his commission? If he takes out his commission, and sends me the rest of the money received, how much should I get?

15. I send 100 doz. of eggs to a commission merchant, which he sells at 30 ¢ a dozen. He takes out his commission and sends me the balance. How much do I get if he charges 2 % commission?

16. How much will I receive for a debt of $ 250, if I pay the lawyer a commission of 5 % for collecting it for me?

17. How much do I receive for 50 bbl. of apples if they are sold by an agent at $ 2.50 per barrel, and he charges me 3 % for his commission?

FIRE INSURANCE

For a certain amount on each $1000 of insurance, or for a certain rate per cent on the value insured, firms will agree to make good any damage which you may suffer from loss by fire.

The written contract between the firm and the one insured is called a **policy**.

The sum which you pay for insurance is called the **premium**.

157. 1. Find the premium for insuring a house for $4000 for three years, if the rate of insurance is $\frac{1}{2}$ % for the time.

$4000 × .005 = $20 premium *Ans.*

2. Find the cost of insurance on a house for $3000 for 3 yr., if the rate of insurance is $5 on each $1000 for the time.

3. Find the cost of insuring the furniture in a house for $1500 for 3 years, the rate of insurance being $5 on a thousand for the time.

4. Find the cost of insuring a house for 3 years for $1800, the rate being $\frac{1}{3}$ % for 3 years.

Note. It is customary to insure for 3 or 5 years, and the rate is quoted for the full time, and not at so much for each year.

5. A factory valued at $45,000 is insured for $\frac{2}{3}$ of its value at 2%. What is the premium?

$45,000 × $\frac{2}{3}$ = $30,000 amount insured.
$30,000 × .02 = $600 premium *Ans.*

Note. Insurance companies do not insure for the full value of the property. Why not?

Where there is great danger of fire, as in a factory, the rate of insurance is high, while in property where the danger of fire is less, the insurance rate is much lower.

6. Find the cost of insuring a house worth $7500 for $\frac{3}{4}$ of its value at the rate of $4 per $1000.

7. What will it cost to insure a house worth $8000 for $\frac{5}{8}$ of its value at the rate of $\frac{1}{2}$ % for insurance?

8. Find the cost of insuring a steam saw mill for one year, the mill being worth \$3000, and insured for $\frac{1}{2}$ of its value at the rate of 4 % for one year.

9. Find the cost of insuring a brick store for \$2000 at $\frac{1}{4}$ %.

10. Find the cost of insuring a wooden store for \$3000 at the rate of \$4 on \$1000.

11. Find the cost of insuring a factory for \$5000 at the rate of $2\frac{1}{2}$ % for one year.

Note. There are many other kinds of insurance, such as insurance against loss by lightning, damage by cyclones, or storms, damage or loss of goods in shipment, life insurance accident insurance, insurance against loss by sickness, etc.

It is not difficult to find the cost of any kind of insurance, after you read the terms of the policy.

Other lines of insurance and other applications of percentage are left for consideration in the advanced book of this series.

158. SUGGESTIONS ABOUT THE SOLUTION OF PROBLEMS

1. Make an abstract of the problem, showing what is given and what is required, writing similar terms under each other and using the equation form of arrangement.

2. See in what terms the answer must be expressed to answer the main question of the problem, and mark the name of the terms for the answer, leaving blank for the numbers. For example: —— lb., —— bushels, \$——, —— sq. ft., etc.

Then perform the necessary operations to answer the question how many, using the numbers in abstract form, and in the most convenient arrangement for computation.

3. Use diagrams freely to picture the problem clearly.

4. When the given numbers are large or strange in form, substitute small numbers, not changing the wording of the problem. By use of these small numbers, fix the steps in the solution, and the necessary computations, then take those steps and make those computations with the given numbers.

5. When there are several operations in a problem, it should be divided into steps, and the question and answer for each step shown clearly in the plan of solution.

6. Whenever the terms of the problem permit you to make a general estimate of the answer, it is well to do so. For example: The answer will be more (or less) than —— and about ——.

This practice avoids absurd answers which may be due to some mistake in computation, and which might pass unnoticed, if you made no estimate of about how much the answer should be.

7. If you find trouble with any problem given below, turn to the page where the subject is treated, and study the development of the topic until you fully understand it.

PROBLEMS IN REVIEW

159. 1. How much should a man receive for a barrel of apples containing 2½ bushels, and sold at 10 ¢ a half peck?

2. Find the cost of 247 pk. of berries at 6 ¢ a quart.

3. Find the cost of 6 dozen oranges at the rate of 3 oranges for 10 ¢.

4. A field is 80 rods long and 25 rods wide. How many rods of fence will it take to inclose it?

5. A rope 13 yd. 2 ft. 6 in. long was cut into 6 equal pieces. How long was each piece?

6. If ⅜ acre produce 60 bushels of potatoes, how many bushels will 3⅜ acres produce at the same rate?

7. A cask of wine contained 58 gal. 3 qt. 1 pt. How much was the wine worth at $1.50 a gallon?

8. A rectangular field contains 360 sq. ft. It is 45 ft. long. How wide is it?

9. Find the cost of 6000 cigars at $4.50 per C.

10. The Civil War began April 11, 1861, and ended April 9, 1865. How long did this war last?

11. The Revolutionary War began April 19, 1775, and closed Jan. 20, 1783. How long did it last?

12. A bicycle marked $50 was sold at a reduction of 12½ %. What was the selling price?

13. A merchant bought $850 worth of goods, and received 25 % and 10 % off. What did the goods cost him?

14. An agent collected 20 % of an $800 debt, and charged 5 % commission on the amount collected. What was his commission?

15. If I borrow $50 at 8 % on Feb. 8 of this year, how much will be due on May 2 of next year?

16. It takes about 35 cu. ft. of space in a bin to hold one ton of coal. How many tons of coal will a bin 12 ft. long, 5 ft. wide, and 5 ft. deep hold?

17. How many bushels of oats will a bin hold which is 9 ft. by 5 ft. by 3 ft.?

18. When cotton is worth 7 ¢ a pound, what is the value of the crop produced on 10 acres, averaging 250 lb. to the acre?

19. At the rate of a mile in 2 minutes, how far will a train go in $2\frac{1}{2}$ hours?

20. At $75 an acre, what is a rectangular field worth that is 160 rods long and 90 rods wide?

21. What would it cost to inclose a field 150 yards long and 80 yards wide by a wire fence at 25 ¢ a foot?

22. If it is 18 feet 8 inches around a square, how long is each side?

23. A bicycle wheel is 26 inches across. How far is it around the wheel?

24. How far does a bicycle move to turn its 26-inch wheel around 10 times?

25. Change 527 pints to bushels, pecks, quarts, and pints.

26. Change $\frac{3}{4}$ inch to the fraction of a foot.

27. A man pays $1000 a year rent for a house. What is his rent for $2\frac{1}{2}$ months?

28. Change $\frac{7}{8}$ to decimal form.

29. How long must a field be that is 20 rods wide to contain 3 acres?

30. What is the cost of 18 dozen notebooks at 32¢ each?

31. What is the circumference of a circle which is 10 inches in diameter?

32. What is the area of a circle which is 4 feet in diameter?

33. What is the cost of 18 gal. 3 qt. of milk at 32¢ a gallon?

34. How many board feet of 2-inch plank will be required to floor a platform 20 feet by 30 feet?

35. Allowing .8 of a bushel to a cubic foot, how many bushels of grain will a bin hold whose dimensions are 4 ft. by 3 ft. by 4 ft.?

36. In a race one boat gains upon the other 5 ft. in every 50 yards. How far ahead will it be at the end of a mile race?

37. How long from the surrender of Cornwallis, Oct. 19, 1781, to the battle of New Orleans, Jan. 8, 1815?

38. What part of a bushel is 6 quarts?

39. What will 72 sheets of paper cost at 15¢ a quire?

40. If $\frac{2}{3}$ of a store is worth $2400, what is .3 of a store worth at the same rate?

41. Bought 288 barrels of flour for $1728, and sold it at a profit of $576. What did I get a barrel for the flour?

42. A man having $95 bought 15 sheep and had $20 left. What did he pay for each sheep?

43. What is the interest on $500 for 9 months at 5%?

44. What is the bank discount and the proceeds of a note for $500, when discounted for 90 days at 6%?

45. What will it cost to insure a cargo of goods for $40,000 at $2\frac{1}{2}$%?

46. If 4 bushels of beans cost $12.56, what will 9 bushels cost?

47. Find the difference between $84\frac{1}{3}$ and $42\frac{7}{8}$.

48. Multiply 9008 by 7080 and divide the product by 600.

49. Find the cost of 7 lb. 12 oz. of cheese at 12¢ a pound.

50. What is $\frac{1}{9}$ of 28 bu. 3 pk. 7 qt.?

51. If 5 chairs cost $80, what will 7 chairs cost?

52. The difference between 144 and 24 is how many times 15?

53. How many pints are there in .875 gallon?

54. After losing $\frac{8}{9}$ of his money, a man had $7.50 left. How much had he at first?

55. Divide 4.5006 by .015.

56. What will be the rent of a house for 1 yr. and 3 months at $50 a month?

57. At $6 a ton, what will 750 lb. of coal cost?

58. What decimal of a bushel is 3 pecks?

59. What will 3 dozen eggs cost at the rate of 18 eggs for 25 ¢?

60. Change .375 yard to feet and inches.

61. What will it cost to insure a house for $4000 at $1\frac{1}{2}$ %?

62. If $\frac{5}{8}$ of a pound of tea cost 40 ¢, what will $3\frac{1}{2}$ pounds cost?

63. What is $\frac{3}{4}$ acre of land worth, if $\frac{5}{6}$ acre is worth $60?

64. A tank which holds 400 gallons is $\frac{5}{8}$ full. How many gallons more will it take to fill it?

65. Find the cost of 1460 lb. of hay at $20 a ton.

66. Find the cost of 4 bu. 3 pk. of wheat at $1.20 a bushel.

67. How many acres in a field that is 80 rods square?

68. Bought 150 bbl. of flour at $6.75 a barrel, and sold it at an advance of 10%. What was the selling price?

69. Bought 2500 envelopes at $65 per M at 10% and 5% off. What amount did they cost?

70. Find the commission of a sale of cotton for $6450 at 1½%.

71. Find the cost of insuring a house for $6500 at ¾%.

72. A man owes a note for $600 and interest at 6% for 2 yr. 4 mo. How much should he pay?

73. $600 is due me in 3 months. How much cash should I get for it now, if discounted at 6%?

74. How much wheat will be required to seed 8 acres, allowing 2 bu. 2 pk. to the acre?

75. A man bought .25 of an acre of land for $75. What would 1½ acres cost at that rate?

76. A farmer sold 45½ bushels of wheat for $36.40. What price did he receive a bushel?

77. Write a receipt for rent for $60 for last month, which Mr. Samuel Adams paid you for rent of a house.

78. Find the cost of a cement sidewalk 40 ft. long and 5 ft. wide at 15¢ a square foot.

79. How many cubic yards of earth will it take to raise the surface of a lawn 2 feet, the lawn being 24 ft. long and 18 ft. wide? What will it cost at 75¢ a cubic yard?

80. How many board feet of lumber will I need to cover a board walk 50 ft. long and 4 ft. wide, if I use boards $1\frac{1}{2}$ inch thick?

81. A gardener works 8 hours a day for 6 days at 25¢ an hour. How much does he earn?

82. What is the weight of a can of water filled with 40 quarts of water, if the can weighs 15 lb., and the water weighs 1 lb. to the pint?

83. After selling $\frac{3}{8}$ of his wheat, a farmer has 400 bushels left. How many bushels had he at first?

84. Divide 374,628 by 697.

85. Find the cost of $37\frac{1}{2}$ yards of cloth at 40¢ a yard.

86. Find the cost of 64 yards of cloth at $62\frac{1}{2}$¢ a yard.

87. Find the cost of 18 yards at 98¢ a yard.

88. What will 3 bushels of chestnuts cost at 25¢ a peck?

89. Find the cost of 4800 lb. of coal at $6.50 a ton.

90. What is the interest at 6% on a note for $200 from May 1, 1908, to July 1, 1909?

91. What will it cost to cement a cellar bottom, which is 30 ft. by 25 ft. at 15¢ a square foot?

92. Add $16\frac{3}{4}$, $24\frac{1}{2}$, $18\frac{7}{8}$, $120\frac{1}{3}$, $64\frac{3}{8}$.

93. From $120\frac{2}{3}$ subtract $16\frac{3}{8}$.

94. Divide 120 by $\frac{2}{3}$.

95. Divide $\frac{2}{3}$ by 18.

96. Divide $1\frac{1}{2}$ by $3\frac{1}{4}$.

97. Multiply $\frac{3}{8}$ by 24.

98. Multiply $2\frac{1}{4}$ by $3\frac{1}{2}$.

99. Multiply $160\frac{1}{2}$ by 48.

100. Divide 18 by .009.

101. Divide .8 by 40.

102. Multiply $\frac{3}{4}$ by .125 and give your answer in decimal form.

103. Multiply .875 by $\frac{1}{8}$ and give your answer in fractional form.

104. Add $245.18, $160.75, $134.80, $150.25, $175.40, $160, $750.15.

105. At 6¢ each, how many pencils can you buy for $2.50?

106. Bought $12\frac{1}{2}$ yards of cloth at 50¢ a yard and gave the clerk a $10 bill. How much change should I receive?

107. Find my commission at 2% for selling 100 doz. eggs at 30¢ a dozen.

108. How much will I make on 60 oranges bought at the rate of 2 for 5¢ and sold at the rate of 3 for 10¢?

109. How long a string will I need to wind 50 times around a baseball bat which is 2 inches in diameter?

110. Find the cost of $\frac{3}{4}$ bushel of grain at 3¢ a quart.

111. How many tiles, each 3 inches square, will cover the space around a fireplace, the space being 5 ft. by 3 ft.?

112. How many bushels in 8 boxes of beans, each box containing 20 quarts?

113. Find the amount of $650 from Oct. 4, 1884, to May 6, 1886.

114. How many acres in a field 18 rods long and 12 rods wide?

115. A man weighed his loaded wagon, and then unloaded the hay and weighed the empty wagon. The loaded wagon weighed 3875 pounds. The empty wagon weighed 1425 pounds. What was the hay worth at $20 a ton?

116. How many pounds of water are there in 10 bushels of fresh potatoes, if the water in the potatoes is 78% of their weight? (Allow 60 lb. to the bushel for potatoes.)

117. If wheat produces 83 % of its weight in flour, how many pounds of flour can be obtained from 10 bushels of wheat? (Allow 60 lb. to the bushel for wheat.)

118. At 3¢ a foot, what is the cost of sufficient picture molding to go around a room which is 14 ft. by 17 ft.?

119. A windmill pumps 2 gallons of water a minute. How many gallons will it pump in $3\frac{1}{2}$ hours?

120. A coat has a row of 8 buttons, set $2\frac{1}{2}$ inches apart. How long is the row? (Picture.)

121. Find the cost of insuring a house for $1500, and its contents for $500, at $2.50 per $1000.

122. How many pupils can stand at a blackboard 12 ft. long, if each pupil is allowed 28 inches of space? (Picture.)

123. Two men travel in opposite directions, one at the rate of 35 miles a day, and the other at the rate of 40 miles a day. How far apart will they be at the end of 6 days?

124. Two men start from different places, distant 189 miles, and travel toward each other. One goes 4 miles an hour, and the other 5 miles an hour. In how many hours will they meet?

125. A post stands $\frac{1}{6}$ in the mud, $\frac{1}{4}$ in the water, and 21 feet above the water. What is the length of the post?

126. What is the cost of 18,640 feet of timber at \$4.50 per 100 ft.?

127. What will be the cost of a load of oats weighing 1450 lb. at 40¢ a bushel? (Allow 32 lb. to the bushel for oats.)

128. How many square feet in the area of a triangle whose base is 64 ft. and its altitude 26 ft.?

129. There were 45 pupils registered in a class, and the per cent of daily attendance was 80. What was the total attendance for this class for 5 days?

130. From a certain meadow 25 tons of hay were cut last season. If the crop is 10% less this year, how many tons can be cut from this meadow?

131. A man invests \$1500 in the poultry business. If his rate of profit is 15% on his investment, how much does he make?

132. A boy hires a boat at the rate of 25¢ an hour. He starts at 10.15 A.M. and returns at 1.30 P.M. How much should he pay?

133. How many yards a second does a train move which is running at the rate of 40 miles per hour?

134. A train running from New York to Philadelphia, a distance of 90 miles, makes the whole distance in 1 hr. 35 min. What is the rate per hour?

135. What is the value in United States money of 1000 francs, if each franc is worth .193 dollar?

136. How much will it cost to dig a cellar 36 ft. long, 28 ft. wide, and 6 ft. deep at $.45 per cubic yard?

137. What is the cost of 2 sticks of timber, each 30 ft. long, 10 in. wide, and 10 in. thick, at $30 per 1000 board feet?

138. What will it cost to pave a street 300 ft. long and 60 ft. wide at $.25 per square foot?

139. A man bought 1320 acres of land, and sold $37\frac{1}{2}$% of it. How many acres had he left?

140. If a man's salary is $1350 a year, and his expenses $87\frac{1}{2}$% of that amount, how much can he save yearly?

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